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# ROYAL METEOROLOGICAL SOCIETY

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QUARTERLY JOURNAL
OF THE
ROYAL METEOROLOGICAL SOCIETY

Vol. 76 JANUARY, 1950 No. 327

## EDITORIAL

It would be improper to allow the *Journal* to appear in a new style without a few words of introduction, something to ease the embarrassment of a bold break with tradition, which some may think an unseemly following of fashion. We share with many an affection for the old page, the handy number which slipped undamaged into the greatcoat pocket. Also we know that it has proved adequate for many communications, now classical, from eminent authorities whose style might be emulated, with profit, by many a young research worker — hurrying into print. But there has certainly been a change of emphasis, leading to some inconvenience with the narrow page ; diagrams have become more popular, and more elaborate on the whole, while the mathematical expressions, which find their way into almost everything today, seem to grow ever longer. If the new appearance is admired we admit a desire to look well, if it is not, we claim it as strictly functional.

Coinciding, by the Council's decision, with the Centenary Year of the Society, the adoption of the new format prompts reflexion upon the place which the *Journal* has taken and should take in the literature of our science. While looking, with pride, at the important contributions to knowledge published over the many years we cannot escape the fact that the status of meteorology has been transformed in quite recent years. Under the forced stimulus of two wars it has emerged, from a long period of slow growth in rather a quiet corner of natural science, as a mature member of the community, acknowledged by state and industry as having a significant contribution to make to the general economy ; our universities show increasing interest, with Sir David Brunt's Department of the Imperial College as an inspiring lead.

The number of meteorologists in the country engaged in full time research or holding staff appointments in the universities has increased several fold in the last decade. This Society must welcome the change but must also realise its responsibilities. The results of research must be published and we believe that our many non-professional Fellows will be the first to say that the *Journal* is the proper medium for everything that is best in British meteorology — even if more and more of it becomes mathematical physics readable only to the expert. The present Editor, now due to give way to a successor, makes no claim to initiative, has needed none, in these three years of office : the task has been merely to publish what was deemed worthy of publication. But, we believe, the volume of good work will increase and the pressure on the *Journal* will soon become embarrassing ; we gladly leave the solution to our successor but can see the *Journal* settling down to a sober eminence amongst the best scientific journals — worthy and distinguished — and what an inspiration was *Weather* !

## CENTENARY CELEBRATIONS OF THE SOCIETY

28 March to 3 April 1950

Fellows and oversea readers will for the most part be already informed of the activities which have been arranged to mark the centenary of the foundation of the Society, 3 April 1850. The opening meetings are to be held in Oxford, with the hospitality of Christ Church, and will take the form of scientific symposia opened by well known authorities, many from amongst our foreign guests. The subjects chosen for the symposia, which open on Tuesday 28 March 1950 and continue for four days, are :

*p.m. Tuesday 28 March*
Radiation and its effect on the troposphere and lower stratosphere.

*a.m. Wednesday 29 March*
Physics of clouds and precipitation.

*p.m. Wednesday 29 March*
Structure of weather systems.

*a.m. Thursday 30 March*
General circulation of the atmosphere.

*p.m. Thursday 30 March*
Climatic change.

*a.m. Friday 31 March*
Meteorology and the community.

There is already promise of much support from many countries and it is hoped that the meetings will be of real significance in the progress of our science. It will fall to the *Journal* to publish the papers presented and these will form the material of a centenary number later on in the year.

The further activities, then moving to London, will include social functions, and visits of scientific interest.

The Director of the Meteorological Office has arranged a visit to the Headquarters establishment of the Meteorological Office at Harrow, where the world-famous Meteorological Library and the Instrument, Marine and Climatological divisions are housed. In the evening a conversazione will be held in the Society's rooms.

On the morning of 3 April, Professor Sir David Brunt and his staff will be at home in the Department of Meteorology, Imperial College, when the problems of atmospheric turbulence will be discussed. This will be followed by the Centenary meeting in the Society's rooms and the Centenary dinner to be held at the Connaught Rooms.

The Science Museum is to mark the occasion by mounting a meteorological exhibit on a comprehensive scale. We also understand that meteorology will receive special attention during the period in talks from Broadcasting House and articles in the national press. Altogether there is promise of enjoyable, stimulating and valuable activity.

The *Journal* takes this opportunity of welcoming in advance all the Fellows, Foreign Members and guests who may find it possible to join the celebrations.

551.547 : 551.557

# A study of the deviations of wind speeds and directions from geostrophic values

By W. L. GODSON, M.A., Ph.D.

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(Manuscript received 26 September 1949)

SUMMARY

From a knowledge of the three-dimensional wind field as a function of space and time, the total wind acceleration and the trajectory curvature can be computed, permitting the evaluation of the geostrophic wind vector without recourse to analysed isobars or height contours.

A study of 618 cases at 10,000 ft for a 3-day November sequence, using observed winds over the North American continent, revealed that the mean absolute angle between the reported wind and the geostrophic wind directions was approximately 15°. Large deviation angles were associated with low geostrophic wind speeds, and thus indirectly with low latitudes. The corresponding mean cross-isobar wind component was slightly greater than 5 mi $hr^{-1}$. The mean absolute deviation between wind and geostrophic wind speeds was of the order of 7 mi $hr^{-1}$, the corresponding mean relative deviation being just under 30 per cent. Large relative deviations were associated chiefly with low geostrophic wind speeds, hence indirectly with low latitudes.

The mean total wind acceleration slightly exceeded 40 mi $hr^{-1}$ $day^{-1}$. The mean trajectory curvature corresponded to a radius of curvature of 175 mi. It was found that the mean components of these terms associated, formally, with horizontal and vertical advection were virtually identical in magnitude, illustrating the importance of the vertical velocity contribution to wind accelerations and curvatures. The mean local tendency component was numerically smaller than the two advective terms in the case of accelerations, but numerically larger in the case of trajectory curvatures.

## 1. INTRODUCTION

It is well realized by most meteorologists that a complete knowledge of the field of pressure in the atmosphere does not imply a complete knowledge of the field of horizontal (or vertical) motion, although this latter knowledge would be highly desirable. Standard methods of computation of winds from pressure data yield results which are little better than geostrophic winds, as has been demonstrated by Neiburger *et al.* (1948). That report showed that part of this error was ascribable to errors of personal estimate resulting from variations in analyses of upper level charts. The remainder of this error is ascribable to factors influencing the wind speed and direction which cannot be obtained from pressure data, such as the total acceleration and a vertical velocity contribution to the trajectory curvature. The present report will investigate these factors and will attempt to assess their magnitudes.

## 2. THEORY

The equations for the horizontal component of the wind vector can be expressed in the form :

$$\dot{V} = l\, V_g \sin \alpha \qquad (1)$$

$$V = V_g \cos \alpha - \frac{V^2\, K_T}{l} \qquad (2)$$

In these equations $V$ is the horizontal wind velocity, $V_g$ the geostrophic wind velocity, $\dot{V}$ the horizontal wind acceleration (*i.e.*, along the trajectory), $K_T$ the curvature of the trajectory, $l$ the Coriolis parameter ($2\omega \sin \phi$), and $\alpha$ the angle from the geostrophic wind direction to the actual wind direction (considered positive when measured in a counterclockwise sense).

If the $\tau$ -axis is chosen tangential to the actual wind direction the wind acceleration, $\dot{V}$, may be expressed as :

$$\dot{V} = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial \tau} + w \frac{\partial V}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this equation $t$ represents time, $w$ the vertical velocity, and $z$ the vertical co-ordinate.

If $\psi$ represents the wind direction (measured in a counterclockwise sense from some reference direction) the trajectory curvature, $K_T$, can be expressed as

$$K_T = \frac{d\psi}{d\tau} = \frac{\dot{\psi}}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The individual time derivative, $\dot{\psi}$, may be expanded as

$$\dot{\psi} = \frac{\partial \psi}{\partial t} + V \frac{\partial \psi}{\partial \tau} + w \frac{\partial \psi}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If Eq. (5) is substituted into (4), and $\partial\psi/\partial\tau$ replaced by $K_s$, the local or streamline curvature, there is obtained

$$K_T = K_s + \frac{1}{V} \frac{\partial \psi}{\partial t} + \frac{w}{V} \frac{\partial \psi}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The final form of the equations of horizontal air motion is obtained by substitution of Eq. (3) into (1) and (6) into (2), giving

$$V_g \sin \alpha = \frac{1}{l} \frac{\partial V}{\partial t} + \frac{V}{l} \frac{\partial V}{\partial \tau} + \frac{w}{l} \frac{\partial V}{\partial z} \quad . \quad . \quad . \quad . \quad (7)$$

$$V_g \cos \alpha = V + \frac{V^2 K_s}{l} + \frac{V}{l} \frac{\partial \psi}{\partial t} + \frac{wV}{l} \frac{\partial \psi}{\partial z} \quad . \quad . \quad (8)$$

## 3. Experimental procedure

All the terms on the right hand sides of Eqs. (7) and (8) can be evaluated from a knowledge of the horizontal and vertical components of air motion. Thus Eqs. (7) and (8) can be solved for $V_g$ and $\alpha$, obviating the necessity of constructing the pressure field objectively without using reported wind data. In addition the magnitudes of the various terms involved in Eqs. (3) and (6) can be studied, and in particular the effect of the contributions due to vertical motion.

This procedure was applied to the 10,000 ft data, at 12-hourly intervals, from 23 November, 0330 G.C.T., to 26 November, 0330 G.C.T., 1944. This period was chosen more or less at random for the determination of vertical velocities and with respect to winds was probably fairly representative of conditions in the winter half of the year. The computations were carried out with all upper wind data for North America, 618 cases in all, although most of these were from the

continental United States. At 6-hourly intervals 10,000 ft wind charts were plotted and analyzed by the construction of streamlines and iso-velocity lines. The charts at the intermediate hours, between periods of radiosonde information, were used to maintain continuity of analysis and to obtain interpolated values of $\psi$ and $V$ for the computation of the local tendencies $\partial\psi/\partial t$ and $\partial V/\partial t$. These local tendencies were obtained from data, observed or interpolated, over a 24-hour interval. An attempt was made, by both graphical and analytical means, to obtain a tendency which would be reasonably representative of an instantaneous value. The streamline curvature ($K_s$) and the local trajectory acceleration ($V\ \partial V/\partial\tau$) were obtained directly from the charts showing streamlines and iso-velocity lines. The shears with height of the wind speed ($\partial V/\partial z$) and direction ($\partial\psi/\partial z$) were evaluated from plotted wind hodographs, and again attempts were made to obtain derivatives applicable to the 10,000 ft level. The fields of vertical velocity ($w$) were obtained by a method based on the integrated equation of continuity, similar to that proposed by Exner (1917). These values were instantaneous values, being based on the distribution of the horizontal velocity components.

This procedure for the evaluation of ageostrophic components involves many sources of error but has the advantage that it determines *directly* a relatively small vector wind difference through the computations of the terms involving $\dot{V}$ and $K_T$. It was felt that a statistical study of the 618 cases computed would give a reasonable picture of the deviations, in both speed and direction, of the true wind from the geostrophic wind, as well as a reasonable estimate of the relative contributions of the time tendency and horizontal and vertical advective terms to the wind acceleration and trajectory curvature.

## 4. Wind accelerations and deviations from the geostrophic wind direction

For all the 618 cases the mean absolute deviation of the wind direction from the geostrophic, with its standard error, was :

$$a = 14{\cdot}4 \pm 0{\cdot}7°.$$

In other words, the true wind is about as likely to blow at an angle of more than 15° to the right or left of the contours as it is to blow at an angle of less than 15°. Studies of the deviation of the wind direction from the geostrophic have been made in the past but it has usually been difficult to separate the experimental errors of estimate from the true deviations. Seeliger (1937) conducted a most objective and comprehensive examination of the deviations. His results showed the typical Ekman spiral with the top of the frictional layer at approximately 1·5 km. Above this level the algebraic mean deviations were approximately zero, and the arithmetic mean of the directional deviations for all levels studied was 10·4 ± 0·7°. Seeliger excluded from his study low wind velocities which was not done in the present investigation. When this factor is taken into account the mean deviations become highly comparable, since low wind velocities were found to be associated with much larger deviations than normal velocities. A very similar study was undertaken recently by Bannon (1949) who found, for Liverpool, absolute mean values

of the deviation angle of 10·0° at 3,000 ft, 9·2° at 700 mb and 9·8° at 500 mb. Bannon also excluded low wind velocities from his study.

The problem has been investigated in a slightly different manner by Miller (1948) who evaluated the total acceleration directly from 12-hr isentropic trajectories. His mean value of the deviation angle was 8°, this lower value arising in part from a much greater smoothing of wind data than employed by other investigators.

The variation of this deviation angle with latitude may now be studied. From Eq. (1) it can be seen than the sine of the deviation angle is directly proportional to the total acceleration and to the cosecant of the latitude and inversely proportional to the geostrophic wind speed. In Fig. 1 the absolute mean value of the sine of the deviation angle, for various latitude ranges, is plotted as a function of the cosecant of the latitude. The vertical line segments on this figure represent the range from the mean minus the standard error to the mean plus the standard error. The probability is approximately 2:1 that the mean lies within this range, assuming that the cases studied constitute a representative sample.

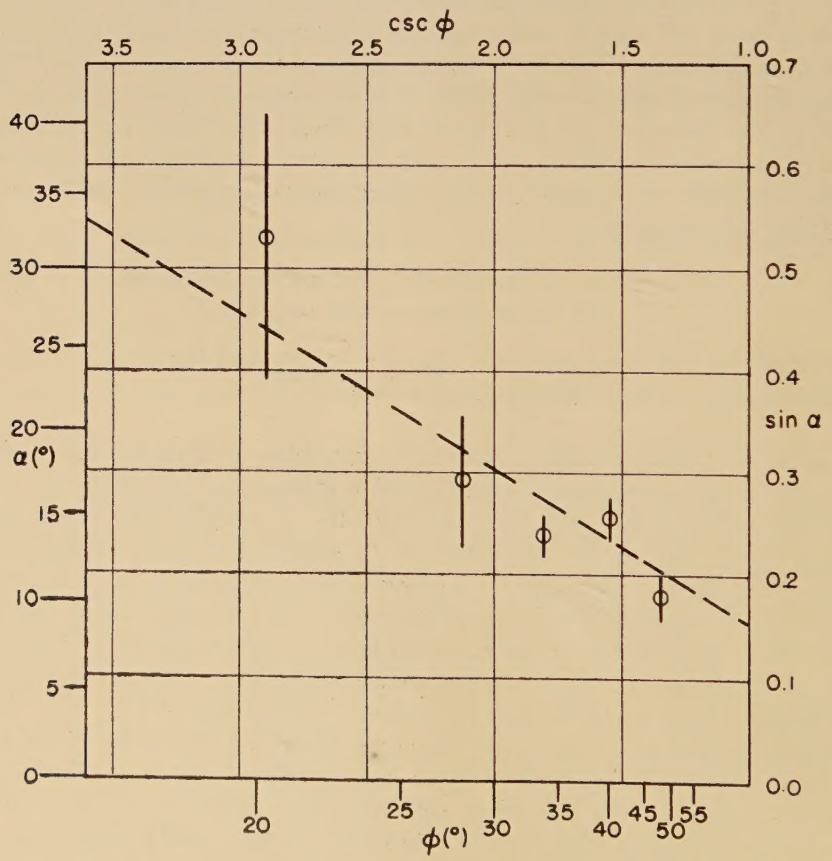


Figure 1. Mean deviation angle as a function of latitude.

There were relatively few cases below latitude 30°, which partially explains the large standard errors of means south of 30°N. The dashed line represents the relation that would be expected if $(\dot{V}/V_g)$ were a constant ; this line passes through the point corresponding to the mean deviation angle (14·4°) and the mean latitude (37·5°) for all cases. It can be seen that the theoretical effect of latitude on the deviation angle is verified, the lower geostrophic wind speeds at low latitudes being roughly compensated for by smaller values of the wind acceleration.

The cases were also subdivided with respect to geostrophic wind speed ranges and the resulting mean deviation angles are shown in Fig. 2 in the same manner as in Fig. 1, the sine of the deviation angle being plotted against the reciprocal of the geostrophic wind speeds. For geostrophic wind speeds less than 35 mi $hr^{-1}$ the deviation angle decreases as the geostrophic wind speed increases (the relation predicted by Eq. (1) ) ; for higher geostrophic wind speeds the deviation angle increases with geostrophic wind speed.

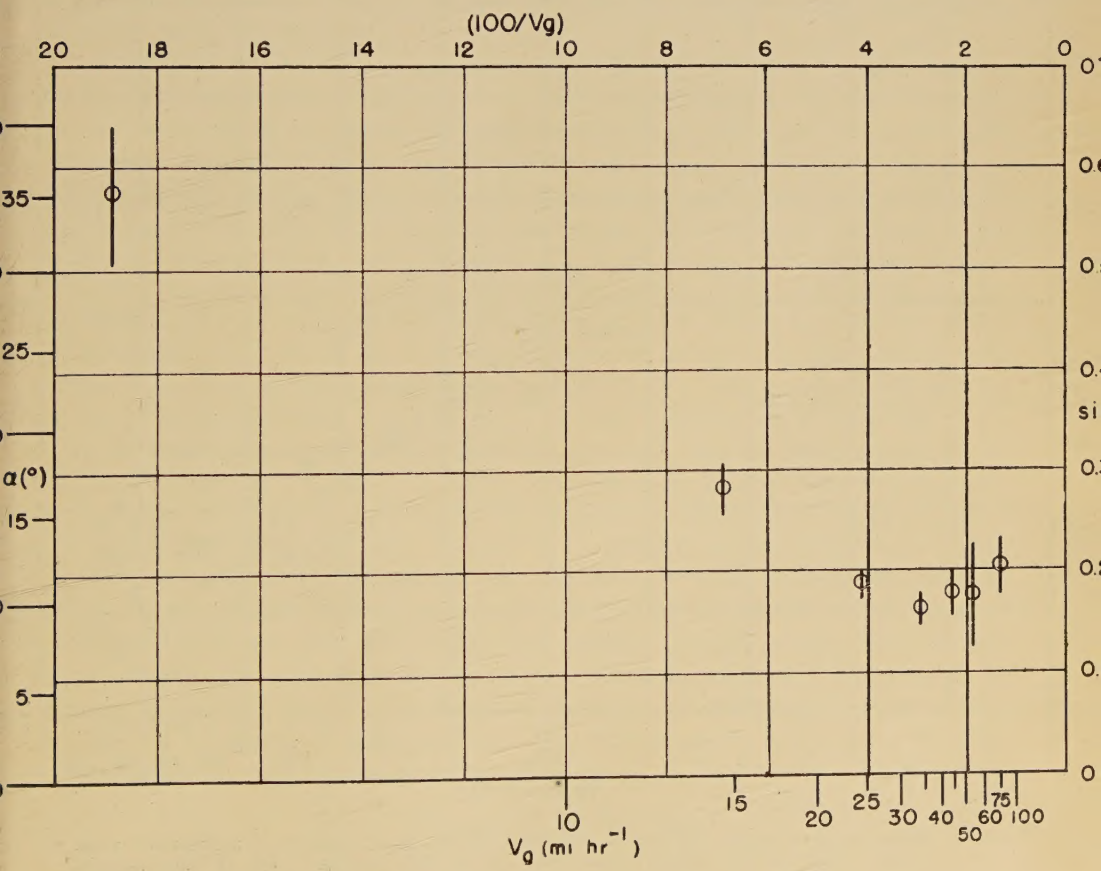


Figure 2. Mean deviation angle as a function of geostrophic wind speed.

Since the mean latitude for each of the geostrophic wind speed groups in Fig. 2 was not the same, the mean deviation angles were reduced to a standard latitude ($\phi_0 = 37{\cdot}5°$) by multiplication by the factor ($\sin \phi$ / $\sin \phi_0$). It can be seen from Eq. (1) that this process essentially eliminates variations arising from the pure latitude effect and focusses attention on variations ascribable to different geostrophic wind speeds and associated accelerations. The mean deviation angles, reduced to the standard latitude, are shown in Fig. 3 by open circles. The data were further subdivided into joint ranges of $V_g$ and $\phi$, using larger ranges than employed in the two previous studies to prevent too large standard errors of the means. The mean deviation for each latitude group (and geostrophic wind speed group) was then reduced to standard latitude. The resulting points are plotted in Fig. 3, each latitude group being represented by a specific symbol. There was no consistent latitude variation of the reduced deviation angles but the dependence

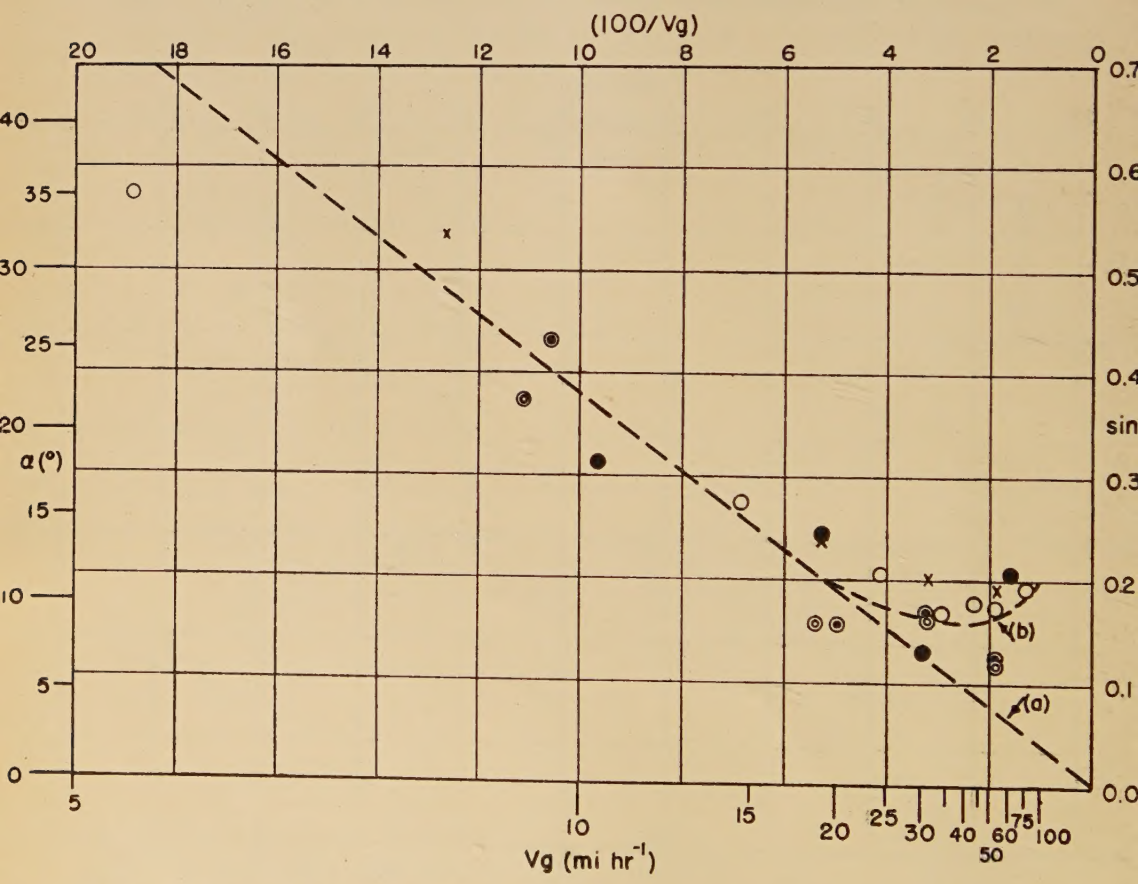


Figure 3. Mean deviation angle, reduced to standard latitude, as a function of geostrophic wind speed. (Open circles denote means from Fig. 2, other symbols refer to fixed latitude ranges as follows : double circles, 17—30N.; closed circles, 30—37N.; crosses, 37—41N.; open circles about closed circles, 41—54N.)

on geostrophic wind speed was well marked. The dashed line (a) was drawn by eye and fits geostrophic wind speeds less than 20 mi $hr^{-1}$. For such geostrophic wind speeds the total acceleration is roughly independent of geostrophic wind speed with a mean value, from Eq. (1) of 29 mi $hr^{-1}$ $day^{-1}$. For geostrophic wind speeds in excess of 20 mi $hr^{-1}$ the total acceleration increases with $V_g$, as may be inferred from the dashed curve (b). For a geostrophic wind speed of 100 mi $hr^{-1}$ the probable absolute mean value of the total acceleration is 153 mi $hr^{-1}$ $day^{-1}$. It is possible that errors inherent in winds of small speed led to the moderate accelerations and relatively large deviation angles for low geostrophic wind speeds.

From Fig. 1 and Fig. 3 the probable magnitude of a deviation angle for any latitude and geostrophic wind speed can be estimated. This angle is likely to be large at low latitudes and for small geostrophic wind speeds ; it is likely to be small for middle and high latitudes and for normal or large geostrophic wind speeds. In the investigation of Bannon (1949) wind speeds less than 18 kt were not included in the analysis so that a mean angle of 10° might be expected at 700 mb at the standard latitude and 7° at the latitude of Liverpool, which compares favourably with Bannon's mean of 9°.

If it is assumed in the computation of winds that the wind direction will be that of the geostrophic wind there will be errors introduced. In the first place the curvature of the streamlines must be assumed to be that of the pressure height contours ; according to Hogben (1946) this introduces an error which will frequently be significant. In the second place the geostrophic wind speed will be used in the computation instead of its component along the true wind direction. The difference between these two quantities, $V_g$ (1 — cos $\alpha$), had a mean value of 0·60 ± 0·03 mi $hr^{-1}$ for the data studied here, and is thus in general negligible. A similar error is introduced in assuming the wind to have the direction of the geostrophic wind. Thus the true wind component along the height contours will differ from the true wind speed by an amount $V$ (1 — cos $\alpha$) which will also be of the order of magnitude of 1/2 mi $hr^{-1}$. A more serious error arises in the cross-contour wind component, which although assumed zero in the computation of gradient winds has a value of $V$ sin $\alpha$. For the cases studied here this quantity had a mean value of 5·2 ± 0·2 mi $hr^{-1}$ and is thus in general not negligible, since the mean wind speed was only 26·6 mi $hr^{-1}$. A flow component towards higher or lower pressure of this magnitude will have an appreciable influence on the field of divergence and convergence and studies of atmospheric models which disregard accelerated flow are open to considerable criticism on this score. For the 618 cases studied the total wind acceleration, $\dot{V}$, had an arithmetic mean value of 41·0 ± 1·9 mi $hr^{-1}$ $day^{-1}$. The standard deviation was 48·0 mi $hr^{-1}$ $day^{-1}$ , indicating the variability of this quantity. It is of interest to study the arithmetic mean values of the component terms of $\dot{V}$, as given by Eq. (3). These were :

$$\frac{\partial V}{\partial t} = 15{\cdot}4 \pm 0{\cdot}5 \text{ mi } hr^{-1} \text{ } day^{-1}$$

$$V\frac{\partial V}{\partial \tau} = 27{\cdot}6 \pm 1{\cdot}3 \text{ mi } hr^{-1} \text{ } day^{-1}$$

$$w \frac{\partial V}{\partial z} = 27{\cdot}6 \pm 1{\cdot}9 \text{ mi hr}^{-1} \text{ day}^{-1}$$

The vertical and horizontal advective terms are of identical orders of magnitude and both are almost twice as large as the local tendency term, in agreement with the findings of Miller (1948).

Houghton and Austin (1946) studied wind component accelerations by evaluating the ageostrophic wind components along X and Y axes. They found mean values of acceleration components, $\dot{u}$ and $\dot{v}$, of approximately 50 mi hr$^{-1}$ day$^{-1}$, which would be expected to be of the same order of magnitude as the arithmetic mean value of $\dot{V}$. They also computed values of the component terms of $\dot{u}$ and $\dot{v}$ and solved for the terms $w\ \partial u/\partial z$ and $w\ \partial v/\partial z$. Such calculations indicated that these terms were at least of the same order of magnitude as the local tendency and the horizontal advective terms, a result which is corroborated by the present study.

## 5. Trajectory curvatures and deviations from the geostrophic wind speed

The deviation of the wind speed from the geostrophic was evaluated in the present study as the non-dimensional quantity $[(V—V_g)/V]$, termed the relative wind speed deviation. In the previous section it was shown that $V_g \cos \alpha$ may be replaced in Eq. (2) by $V_g$ with very little error, and as a result that latter equation can be expressed as

$$\frac{V - V_g}{V} = -\frac{V K_T}{l} = -\frac{\dot{\psi}}{l}, \qquad (9)$$

using Eq. (4) to substitute for $K_T$.

The absolute mean value of the relative wind speed deviation for the 618 cases was found to be :

$$\frac{V - V_g}{V} = 0{\cdot}28 \pm 0{\cdot}01$$

This quantity was computed by Neiburger *et al.* (1948) from a study in which geostrophic speeds were read from analyzed 700 mb charts. Their mean value was 0·35, slightly higher than the value deduced here, ascribable in part to a lower mean wind speed for their cases than for those studied here.

Neiburger *et al.* (1948) also studied the distribution of absolute values of $(V - V_g)$ as a function of latitude to ascertain whether significantly larger deviations would be found at low latitudes, where the geostrophic wind equation is considered a poor approximation. Their results were not at all conclusive, which is to be expected since actual values of $V$ and $V_g$ at low latitudes are relatively small. Therefore, in the present study the mean relative wind speed deviation, $[(V - V_g)/V]$

was plotted as a function of latitude. Other factors being constant, this relative deviation should be inversely proportional to the sine of the latitude. The observed distribution of absolute means for various latitude ranges is shown in Fig. 4, the standard error being illustrated as in Figs. 1 and 2.

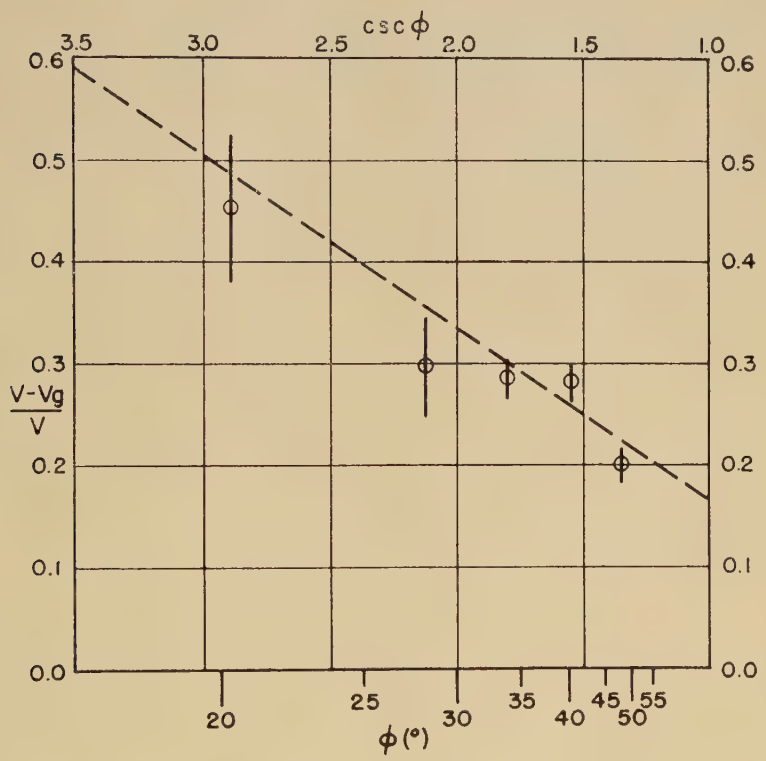


Figure 4. Mean relative wind speed deviation as a function of latitude.

It can be seen from Fig. 4 that larger relative deviations are present at low latitudes than at high latitudes. The dashed curve represents the theoretical relation from Eq. (9), assuming $\dot{\psi}$ to be constant, the curve being adjusted to fit the mean relative deviation and latitude for all cases. This curve fits the observed data reasonably well, suggesting that $\dot{\psi}$ will have comparable values at all latitudes and that the relative deviation does increase markedly as the latitude decreases. This is not true of the actual deviation, $(V - V_g)$, which has a maximum near latitude 35°N., as a result of the higher wind speeds in that zone, and minima at low and high latitudes.

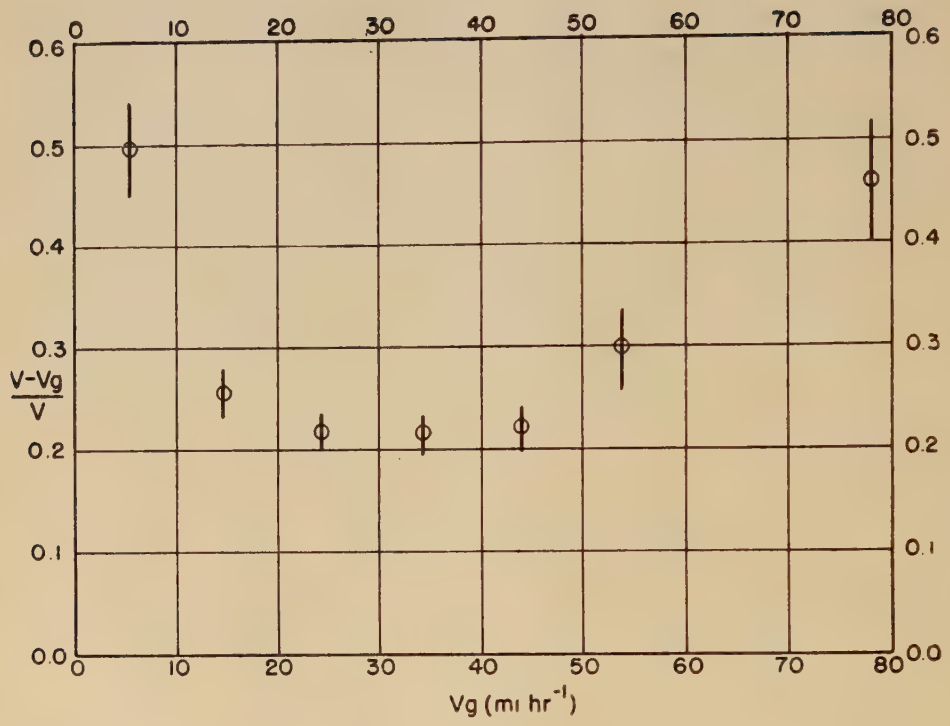


Figure 5. Mean relative wind speed deviation as a function of geostrophic wind speed.

The mean relative wind speed deviation from the geostrophic was also studied as a function of the geostrophic wind speed ; the results of this study are shown on Fig. 5. The data suggest that large values of the relative wind speed deviation are associated with both very small and very large geostrophic wind speeds. In order to remove the effect of latitude on the data in Fig. 5 each value of the absolute mean relative wind speed deviation was reduced to the standard latitude of 37·5° by the factor (sin $\phi$ / sin $\phi_0$) and replotted in Fig. 6 using open circles. The data were further subdivided into joint ranges of $\phi$ and $V_g$, as before, and the means of the relative wind speed deviation, reduced to the standard latitude, are entered on Fig. 6 in the same manner as on Fig. 3. It can be seen that there is a definite tendency toward higher reduced values at the higher latitudes. The straight-line relation (curve (a)) suggests that for $V_g > 40$ mi hr$^{-1}$ the magnitude of the trajectory curvature is virtually independent of wind speed. The fitted curve (b) suggests that for $V_g < 40$ mi hr$^{-1}$ small geostrophic wind speeds are usually associated with small radii of trajectory curvature, assuming that the nature of curves (a) and (b) would be unchanged if the relative deviations were plotted against $V$, rather than $V_g$, so that Eq. (9) could be utilized directly.

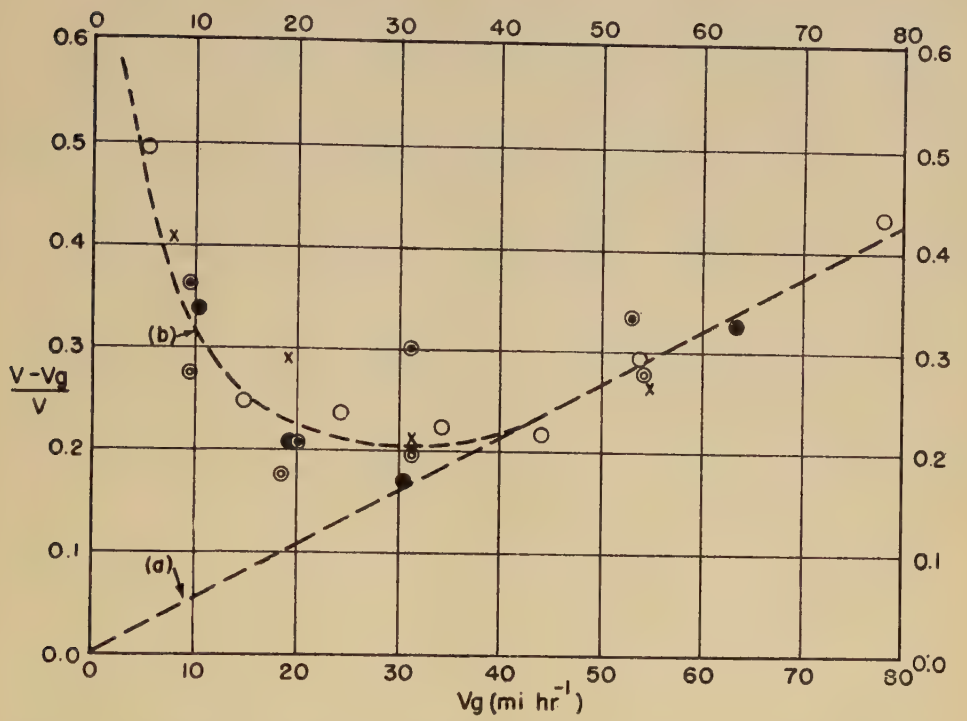


Figure 6. Mean relative wind speed deviation, reduced to standard latitude, as a function of geostrophic wind speed. (Same symbolization as in Fig. 3).

For all cases the absolute mean value of $(V - V_g)$ was 7·4 $\pm$ 0·3 mi hr$^{-1}$, giving from Eq. (9) the mean value of $V^2K_T/l$. The cases studied yielded an arithmetic mean value of $K_T$ of 5·7 $\pm$ 0·4 $(10^3$ mi$)^{-1}$, corresponding to a mean radius of curvature of the trajectories of 175 mi. It is of interest to study the arithmetic mean values of the component terms of $K_T$, the trajectory curvature, as given by Eq. (6). These were :

$$K_s = 3{\cdot}0 \pm 0{\cdot}1 \ (10^3 \text{ mi})^{-1}$$

$$\frac{1}{V}\frac{\partial \psi}{\partial t} = 4{\cdot}1 \pm 0{\cdot}3 \ (10^3 \text{ mi})^{-1}$$

$$\frac{w}{V}\frac{\partial \psi}{\partial z} = 3{\cdot}0 \pm 0{\cdot}3 \ (10^3 \text{ mi})^{-1}$$

Once again the vertical and horizontal 'advective terms' are of identical orders of magnitude but in the $K_T$ relation the local tendency term is a larger term than either of the advective terms, whereas in the $\dot{V}$ expression the local tendency term was significantly smaller.

Some remarks may now be made concerning the experimental findings of Neiburger *et al.* (1948) to the effect that the geostrophic wind equation could not be significantly improved upon in the computation of winds. In the previous section errors arising from the assumption that the true wind direction would be that of the contours were analyzed and it was shown that the cross-isobaric component should not be neglected. However, if this component is to be computed, the complete four-dimensional field of the velocity must already be known (*i.e.*, variations with $x$, $y$, $z$, and $t$), so that it is obviously impossible to make any allowance for total acceleration and cross-isobar flow. In the case of the wind speed equation, Eq. (2), it is necessary to know $\partial\psi/\partial t$, $K_s$, and $w\ \partial\psi/\partial z$. The time term gives the largest contribution to $K_T$ and is also a very difficult term to evaluate accurately. If actual winds are not used it is necessary to evaluate all three terms from motions and properties of contour lines themselves, introducing numerous errors. Furthermore, the term involving the vertical velocity is of the same order of magnitude as the streamline curvature term and vertical velocities will virtually never be available for wind computation, since they too need actual winds for their accurate evaluation.

As a consequence of such considerations it can be deduced that no method available at present will permit of the *rapid* computation of wind velocity vectors with an accuracy significantly superior to the assumption of geostrophic winds, except possibly for large geostrophic wind speeds. It is possible, of course, that methods of successive approximations, such as those outlined by Ertel (1940, 1941) and Van Mieghem (1941), might give more accurate results but the methods are definitely time-consuming and no adequate proof of the convergence of their approximate solutions has as yet been presented.

## 6. Conclusions

(a) For latitudes north of 30° and for geostrophic wind speeds exceeding 20 mi $hr^{-1}$ the angle between the observed wind and the geostrophic wind is of the order of 10°, considerably larger values being possible if these conditions are not fulfilled.

(b) On the average the cross-contour wind component is of the order of 20 per cent of the wind speed. The "isallobaric" contribution to this component is generally smaller than the contributions ascribable to horizontal or vertical "velocity advection."

(c) For latitudes north of 30° and for geostrophic wind speeds between 20 and 45 mi $hr^{-1}$ the difference in speed between observed and geostrophic winds is of the order of 20 per cent of the true wind speed, considerably larger values being possible if these conditions are not fulfilled.

(d) For latitudes south of 30° and/or for low geostrophic wind speeds the geostrophic wind equation gives a poor approximation to the actual wind ; no significant improvement is possible by any existing methods of computing gradient winds.

(e) For latitudes north of 30° and for geostrophic wind speeds from 15 to 50 mi $hr^{-1}$ the geostrophic wind equation gives a fair approximation to the actual wind ; slight improvement may be possible by consideration of contour curvature and the motion of contour systems.

(f) For geostrophic wind speeds exceeding 50 mi $hr^{-1}$ the geostrophic wind equation gives a poor approximation to the actual wind ; at middle and high latitudes a better approximation would be afforded by the gradient wind equation if the trajectory curvature could be evaluated with sufficient accuracy.

## Acknowledgment

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## References


| | | |
|---|---|---|
| Bannon, J. K. | 1949 | *Quart. J.R. Met. Soc.*, London, **75**, p. 131. |
| Ertel, H. | 1940 | *Ann. Hydrogr.*, Berlin, **68**, p. 421. |
| Ertel, H. | 1941 | *Met. Z.*, Braunschweig, **58**, p. 77. |
| Exner, F. M. | 1917 | *Dynamische Meteorologie*, Leipzig and Berlin, p. 71. |
| Hogben, G. L. | 1946 | *Quart. J.R. Met. Soc.*, London, **72**, p. 318. |
| Houghton, H. G. and Austin, J. M. | 1946 | *J. Met. Amer. Met. Soc.*, Boston, Mass., **3**, p. 57. |
| Miller, J. E. | 1948 | *Met. Papers*, New York, **1**. |
| Neiburger, M. *et al.* | 1948 | *J. Met. Amer. Met. Soc.*, Boston, Mass., **5**, p. 87. |
| Seeliger, W. | 1937 | *Beitr. Phys. frei Atmos.*, Leipzig, **24**, p. 130. |
| Van Mieghem, J. | 1941 | *Bull. Acad. R. Belg.*, Cl. Sci. (5), Bruxelles, **27**, pp. 309, 339. |
| Van Mieghem, J. | 1941 | *Mém. Acad. R. Belg.*, Cl. Sci., Bruxelles, **19**, p. 1. |

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# The size distribution of raindrops

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SUMMARY

Experimental data relating to drop size distribution in rain are examined and it is shown that in many cases the size distribution is in accordance with the following formulae

$$1 - F = \exp\,[-(x/a)^n]$$
$$a = A\,I^p$$
$$W = C\,I^r$$

where

$F$ = fraction of liquid water in the air comprised by drops with diameter less than $x$.
$I$ = rate of precipitation.
$W$ = amount of liquid water per unit volume of air.

$A$, $C$, $p$, $r$ and $n$ are constants. If $x$ is measured in mm, $I$ in mm/hr and $W$ in $mm^3/m^3$ the mean values of $A$, $C$, $p$, $r$ and $n$ are 1·30, 67, 0·232, 0·846 and 2·25 respectively. There may be appreciable variations from these mean values, particularly in the case of $n$, if the precipitation is essentially of a showery or orographic nature.

Tables based on these formulae are given showing values of $W$ and of the number and total volume of drops between certain diameter limits for various rates of rainfall.

## 1. INTRODUCTION

During recent years the drop size distribution in rain has acquired increased importance as a result of the use of radar equipment to locate precipitation. The radar signal received from precipitation is exceedingly sensitive to the size of the drops. Ice accretion on aircraft and soil erosion are other spheres in which information on the subject is required and, in addition, the matter has an interest which is purely meteorological. These facts have led a number of investigators to examine drop size distribution during the past six years and the results of their work have considerably augmented the comparatively scanty knowledge of the subject which was previously available.

Inevitably the various workers in the subject have used different methods of presenting their results. The purpose of this paper is to convert the various published results to a common form of presentation and to add the hitherto unpublished results of some similar measurements made in England and Wales.

Drop size distribution may be correlated with many factors such as rate of rainfall, type of rain (continuous, thunderstorm, orographic, etc.), position relative to centre of rainstorm, relative humidity, etc. The volume of experimental measurements available is quite inadequate to examine the influence of all these factors. Moreover some of them are unsuitable for numerical treatment. Accordingly in the following paragraphs drop size distribution has been regarded as a function only of rate of rainfall. This is an over-simplification but since rate of precipitation is itself related to some of the other factors which may affect drop size distribution it is believed that the results of such an analysis may indicate average results even though individual rainstorms may show considerable variations from the average.

In the past there has been discussion about the possibility of raindrop sizes being grouped about certain diameters which bear a simple relation to each other.

The present writer is not convinced that current methods of measuring raindrop sizes are adequately accurate to examine this point and so this aspect of the question has not been considered.

## 2. Methods of measurement

Most of the published results which the present writer has seen have been obtained by one of two methods. The most popular method has undoubtedly been the ' filter paper ' method. This involves exposing to the rain for a short time a sheet of absorbent paper — frequently Whatman No. 1 filter paper — in a frame. The stains made by the drops are rendered permanent by previously treating the paper with a suitable powder dye. Subsequently the stains are counted, measured and interpreted in terms of drop sizes with the aid of a calibration curve appropriate to the paper in use. The second method consists of allowing the drops to fall into pans of sifted flour. The resulting dough pellets are baked and subsequently sized by passing them through variously graded sieves.

No attempt will be made here to describe the experimental technique more fully since reference can be made to the original papers. It may, however, be noted that the labour of reducing the measurements made by the filter paper method is very much greater than that involved in the flour method.

## 3. Formulae for presentation of the results

Owing to the variation of the terminal velocity of a water drop with size the drop size distribution on the ground differs from the drop size distribution in the air. All measurements made so far have been made on the ground and such results are directly applicable to questions of soil erosion. On the other hand for radar and ice accretion problems drop size distribution in the air is required and probably also it is of greater interest as a matter of pure meteorology. Accordingly all results discussed in the present paper are considered in terms of drop size distribution in the air. This is easily obtained from drop size distribution on the ground by dividing the number of drops of a particular size by their terminal velocity. In order to convert results to this form, when necessary, terminal velocities have been taken from a curve based upon the values in Table I.

TABLE I. Terminal velocities of raindrops

| Drop diam. (mm) | 0·5 | 1·0 | 1·5 | 2·0 | 2·5 | 3·0 | 3·5 | 4·0 | 4·5 | 5·0 | 5·5 | 6·0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Term. vel. (cm/sec) | 198 | 397 | 539 | 640 | 723 | 786 | 835 | 872 | 894 | 906 | 912 | 914 |

Most writers have given their results in the form of formulae, or numbers, representing the numbers of drops of different sizes. It appears that the number of drops increases rapidly as the drop size decreases. Thus any curve representing the number distribution of drops will have the highest ordinates corresponding to the smallest drops. For many purposes however the number of drops of a

particular size is less important than the volume of water comprising drops of that size. Accordingly it has seemed preferable to the present writer to concentrate upon the volume of drops of a given size. On account of the diversity of the data dealt with it was also desirable to find a method of presentation which does not depend upon the particular size ranges of drops chosen. For this reason all the data considered have been interpreted in terms of the function $F(x)$ where

$F(x)$ = fraction of liquid water in the air comprised by drops with diameter less than $x$.

It will be shown later that $F$ is frequently of the form

$$1 - F = exp\left[-(x/a)^n\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $a$ and $n$ are constants for a particular rainfall. It will also be shown that whereas $n$ shows no consistent variation with the rate of rainfall the constant $a$ is given by

$$a = A\, I^p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $A$ and $p$ are constants and $I$ is the intensity of precipitation.

If $W$ is the amount of liquid water in unit volume of space it will also be shown that

$$W = C\, I^r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $C$ and $r$ are constants.

Other functions which will be required in the interpretation of the data are

$D_{50}$ = a value of the drop diameter $x$ such that 50 per cent of the water in the atmosphere is comprised by drops with diamater less than $D_{50}$.

$D_{75}$ and $D_{25}$ with definitions analogous to that of $D_{50}$.

$$Z = \Sigma\, x^6 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $x$ is the diameter of a water drop contributing to a radar response and the summation is taken over all the drops in unit volume.

Now if Eq. (1) is true it is easily shown that

$$\left.\begin{aligned} D_{50} &= 0{\cdot}69^{\,1/n}\, a \\ D_{75} &= 1{\cdot}38^{\,1/n}\, a \\ D_{25} &= 0{\cdot}29^{\,1/n}\, a \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and if (2) and (3) are true

$$Z = \frac{6\, W\, a^3}{\pi}\, \Gamma\left(\frac{n+3}{n}\right)$$

$$i.e.,\ Z = Z_0\, I^{r+3p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $$Z_0 = 1{\cdot}91\, C\, A^3\, \Gamma\left(\frac{n+3}{n}\right) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Here $\Gamma$ is the ordinary Gamma Function and $Z_0$ is independent of the rate of precipitation $I$.

The predominant drop diameter, that is the diameter of the drops which account for the greatest volume of water in the air, is the diameter for which $\partial F/\partial x$ is greatest. Denoting this diameter by $D_p$ we easily see that

$$D_p = a \left(\frac{n-1}{n}\right)^{1/n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The fractional volume of water corresponding to drops having diameter between $x_1$ and $x_2$ is clearly

$$exp\ [-(x_1/a)^n] \quad - \quad exp\ [-(x_2/a)^n]$$

and if $x_1$ and $x_2$ are sufficiently close together the number of drops of diameter $(x_1 + x_2)/2$ can be obtained by dividing this quantity by $\pi x^3/6W$ where $x = (x_1 + x_2)/2$ and $W$ is the total volume of water per unit volume of air. An alternative — and more exact — method is to divide $W\ \partial F/\partial x$ by $\pi\ x^3/6$ and integrate the result between the limits $x_2$ and $x_1$. This yields the result

$$\frac{\pi}{6\ W} N\ (x_2\ ,\ x_1) = \gamma\left[\frac{n-3}{n}\ ,\ \left(\frac{x_2}{a}\right)^n\right] - \gamma\left[\frac{n-3}{n}\ ,\ \left(\frac{x_1}{a}\right)^n\right] \quad . \quad . 9$$

where $N\ (z_2\ ,\ x_1)$ is the number of drops with diameter between $x_2$ and $x_1$ and $\gamma$ is the Incomplete Gamma Function.

## 4. The new data

The new data referred to in the introduction consist of measurements made by the filter paper method at Shoeburyness by L. G. Hemens, at Ynyslas (Wales) by A. R. Laird and at East Hill (near Dunstable) by R. F. Jones, all of the Meteorological Office. The same technique was adopted at each of these stations. To the brief details of the method already given in section 2 it is necessary to add only that the filter papers had an effective diameter of 20 cm and that they were exposed for a measured period which varied from a few seconds to one or two minutes according to the rate of rainfall. When analysing the results the numbers of stains corresponding to drops having diameter 0 to ·05 mm, 0·5 to 1·0 mm *etc.*, were counted. The results from each station were grouped separately according to the rate of precipitation (measured by the computed amount of water falling on a known area of filter paper in a measured time). For each group of results the function $F\ (x)$ was evaluated for values 0·5, 1·0, 1·5 mm *etc.* of $x$. Now from Eq. (1) it is clear that

$$\log \log_{10}\left(\frac{1}{1-F}\right) = -0{\cdot}36 + n\ (\log_{10}x - \log_{10}a) \quad . \quad . \quad (10)$$

Thus if Eq. (1) represents the data and $\log \log_{10}\ 1/(1-F)$ is plotted against $\log_{10}x$ the points should lie on a straight line with slope $n$. Furthermore, from the intercept of this line on the $\log \log_{10}\ [1/(1-F)]$ axis, the value of $(0{\cdot}36 + n \log_{10}a)$, and hence of $a$, can be obtained. Fig. 1 shows the points plotted in

this way ($x$ being measured in mm). Except for the smallest and largest values of $F$, and except for the highest rate of rainfall at East Hill, the plots approximate reasonably well to straight lines. In fact we may say that Eq. (1) represents the data fairly accurately if we exclude the 10 per cent of the water comprising the smallest drops and the 5 per cent of the water comprising the largest drops.

In view of the linearity of the lines obtained from other sets of similar data it is probable that the curvature of the line corresponding to the highest rate of rainfall at East Hill is without significance and is to be attributed to the small amount of data corresponding to this line (see Table II). This curvature indicates that Eq. (1) is not applicable to these particular data; since the present analysis is all based on Eq. (1) the data corresponding to the highest rate of rainfall at East Hill will not be considered further.

From the size distribution and the number of drops, the duration of the exposure and area of filter paper and the values of the terminal velocities $W$ has been computed for each group.

Table II indicates the volume of data obtained from these experiments and also gives the values of $n$ and of $a$ obtained from Fig. 1 and the computed values of $W$. We see that there is no consistent variation of $n$ with $I$ but that $a$ increases as $I$ increases. Naturally $W$ also increases with $I$.

In Figs. 2 and 3, $a$ and $W$ have been plotted logarithmically against $I$. The results approximate reasonably well to straight lines. Using the method of least squares Eqs. (2) and (3) have been fitted to each group of data in Figs. 2 and 3, yielding values for the constants as indicated in Table VIII (section 6).

TABLE II. YNYSLAS, SHOEBURYNESS AND EAST HILL DATA

| Station | Range of $I$ mm/hr | Mean $I$ mm/hr | No. of papers | $n$ | $a$ mm | $W$ mm$^3$/m$^3$ |
|---|---|---|---|---|---|---|
| Ynyslas .. | 0·0 — 0·6 | 0·36 | 28 | 3·29 | 1·00 | 30 |
| | 0·6 — 1·8 | 1·32 | 78 | 2·33 | 1·21 | 97 |
| | 1·8 — 3·6 | 2·76 | 96 | 2·43 | 1·36 | 180 |
| | 3·6 — 6·6 | 4·68 | 72 | 2·00 | 1·60 | 270 |
| | 6·6 — 12·0 | 8·88 | 25 | 2·38 | 1·95 | 451 |
| | | Total number of rain periods .. | | | 13 | |
| | | Total number of drops .. | | | 145,123 | |
| Shoeburyness .. | 0·0 — 0·6 | 0·18 | 37 | 1·79 | 1·24 | 13 |
| | 0·6 — 1·8 | 1·14 | 26 | 1·56 | 1·14 | 86 |
| | 1·8 — 3·6 | 2·58 | 24 | 3·02 | 2·11 | 121 |
| | 3·6 — 4·4 | 4·20 | 3 | 2·81 | 2·37 | 169 |
| | | Total number of rain periods .. | | | 6 | |
| | | Total number of drops .. .. | | | 55,984 | |
| East Hill .. | 0·0 — 1·0 | 0·41 | 70 | 2·00 | 1·13 | 31 |
| | 1·0 — 2·0 | 1·42 | 50 | 2·35 | 1·38 | 92 |
| | 2·0 — 3·0 | 2·45 | 10 | 2·11 | 1·73 | 137 |
| | 3·0 — 5·0 | 3·69 | 6 | 1·77 | 2·07 | 183 |
| | 5·0 — 10·0 | 7·44 | 10 | 1·70 | 2·38 | 353 |
| | 10·0 — 35·0 | 25·12 | 3 | — | — | — |
| | | Total number of rain periods .. | | | 18 | |
| | | Total number of drops .. .. | | | 163,317 | |

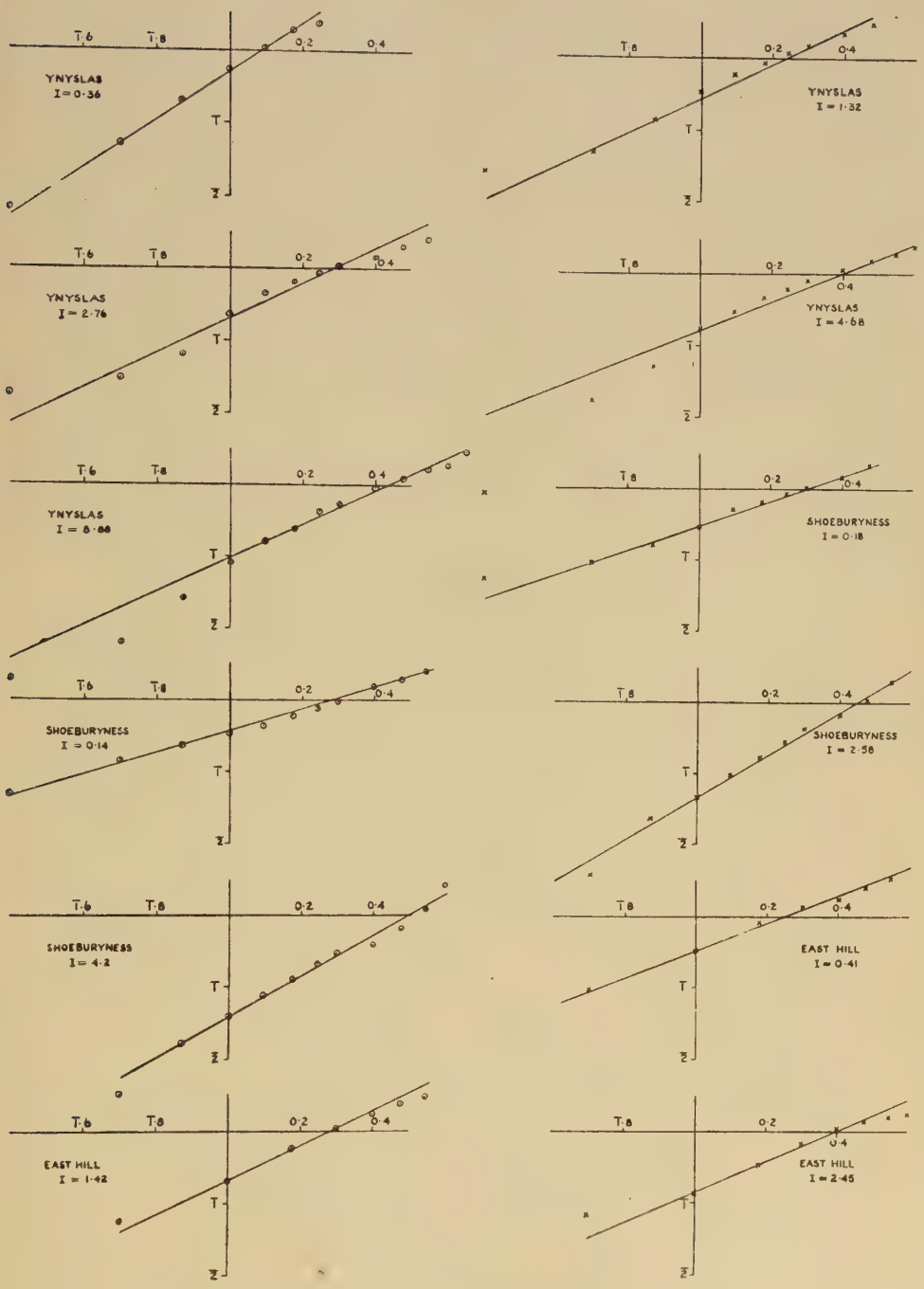


Figure 1a. Fraction of volume of water as a function of drop size.

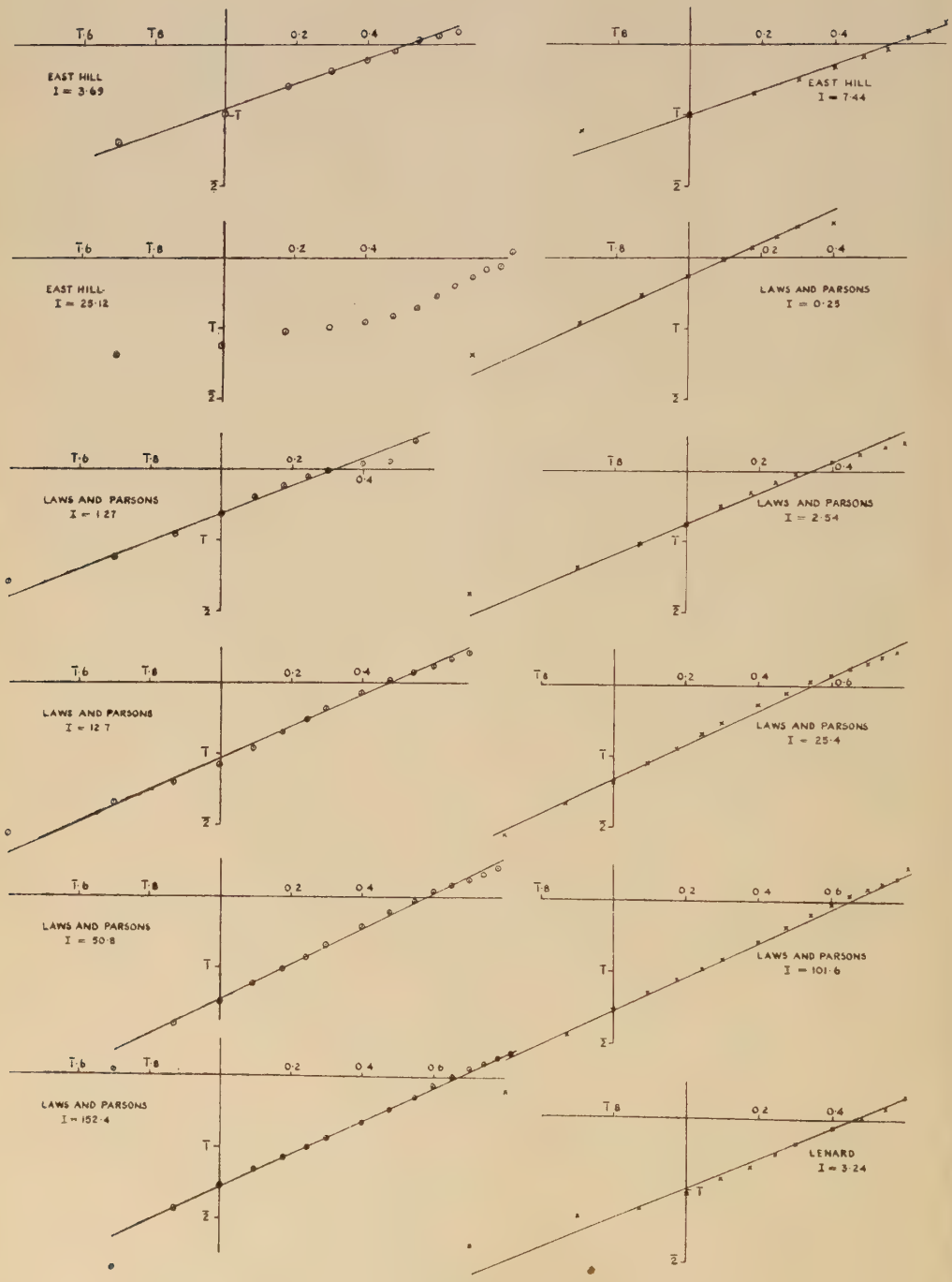


Figure 1b. Fraction of volume of water as a function of drop size.

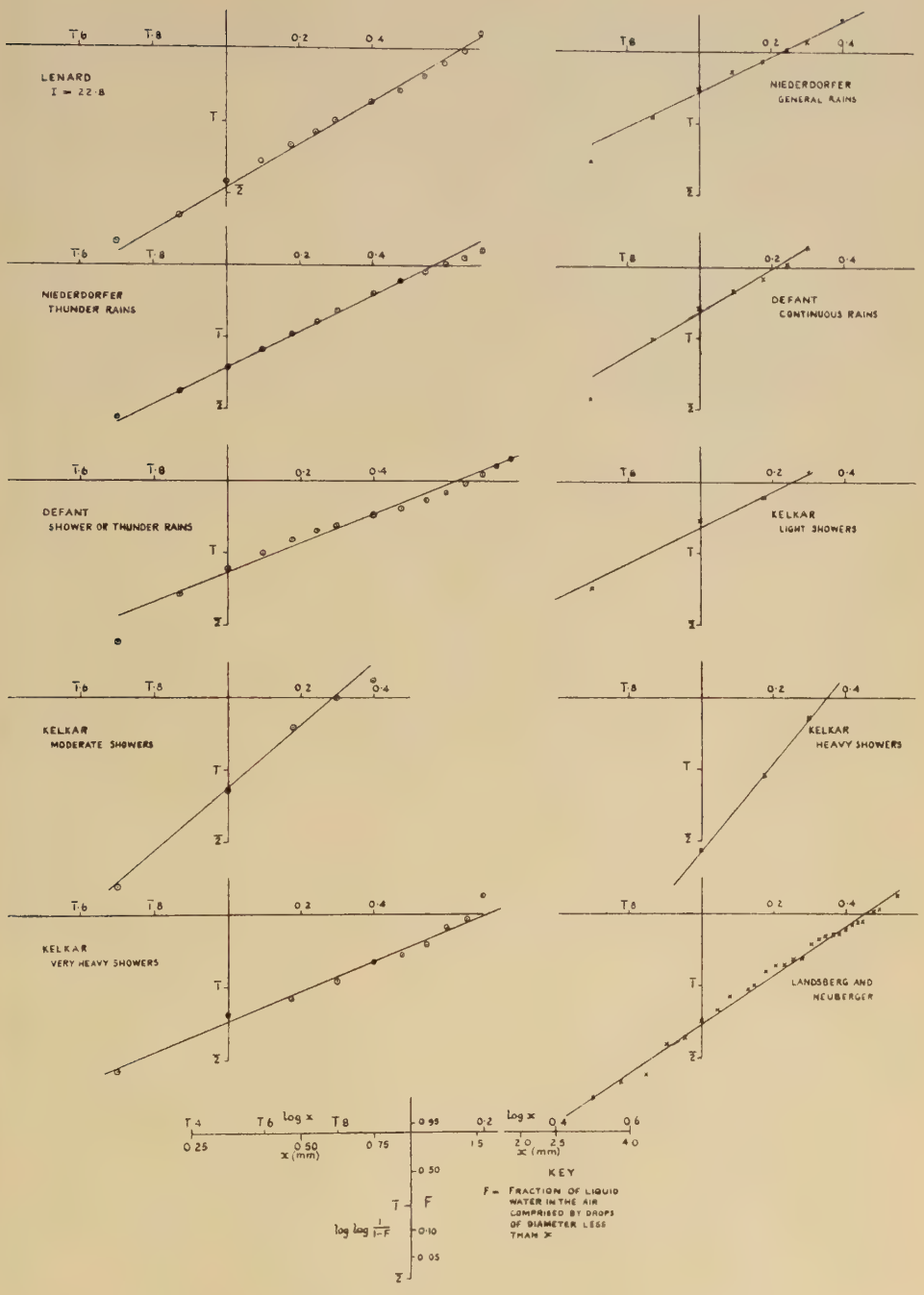


Figure 1c. Fraction of volume of water as a function of drop size.

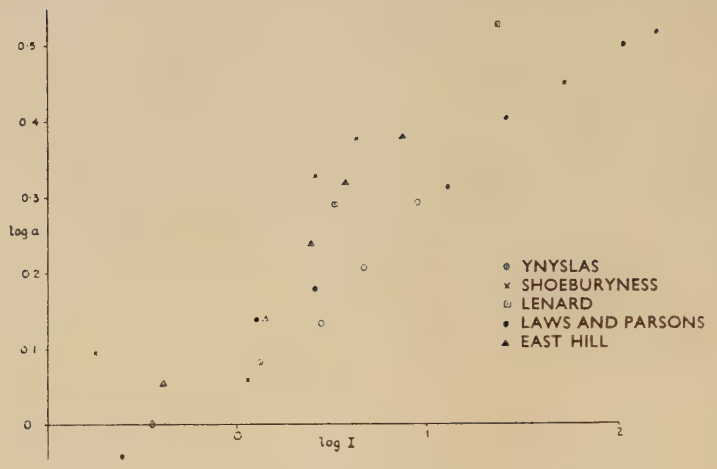


Figure 2. Drop size as a function of rate of precipitation.

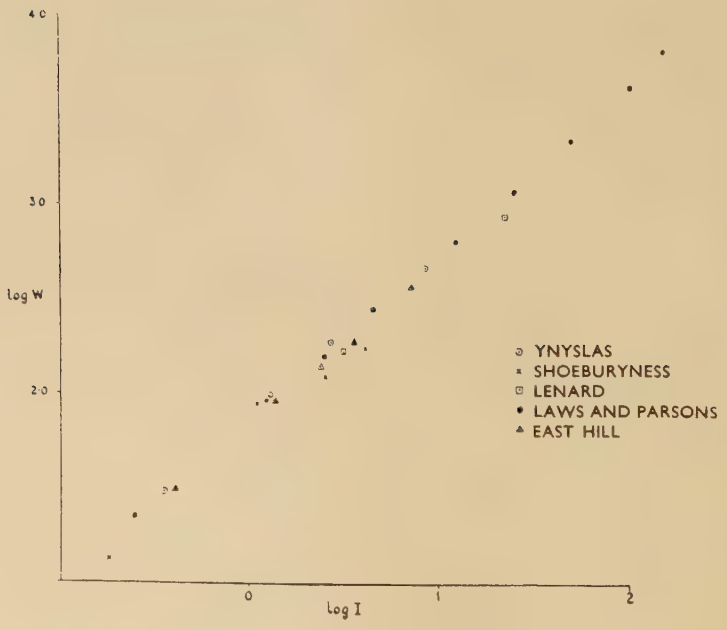


Figure 3. Concentration of liquid water in the air as a function of rate of precipitation.

## 5. Data from other workers

In view of the linearity of the lines in Fig. 1 the data of other workers have been examined to see whether their results can be represented by an equation of the form of Eq. (1). In some cases this could be done only indirectly. In the following paragraphs we shall consider published data on drop size distribution in rain and whether this data can be represented by Eq. (1).

### (*a*) *Laws and Parsons* (1943)

These investigators used the ' flour ' method of measuring drop size distribution. They summarized their results in a table giving the percentage amount of water reaching the ground comprised by drops of different sizes for each of eight different rates of rainfall. There was thus no difficulty in putting the data in a form to test Eq. (1). In Fig. 1 $\log \log_{10} 1/(1-F)$ has been plotted against $\log_{10} x$ and in Figs. 2 and 3 $a$ and $W$ have been plotted logarithmically against $I$. The plots in Fig. 1 show that Eq. (1) accurately represents the data for values of $F$ from 0·1 to 0·95. The values of $n$ and $a$ from the data plotted in Fig. 1 and computed values of $W$ are given in Table III.

TABLE III. Values of $n$, $a$ and $W$ from Laws and Parsons (1943) data

| $I$ mm/hr | $n$ | $a$ mm | $W$ mm$^3$/m$^3$ |
|---|---|---|---|
| 0·25 | 2·26 | 0·90 | 23 |
| 1·27 | 1·96 | 1·37 | 91 |
| 2·54 | 2·19 | 1·51 | 155 |
| 12·7 | 2·24 | 2·04 | 620 |
| 25·4 | 2·44 | 2·50 | 1126 |
| 50·8 | 2·46 | 2·77 | 2110 |
| 101·6 | 2·41 | 3·12 | 4010 |
| 152·4 | 2·34 | 3·23 | 6110 |

The constants in Eqs. (2) and (3) have been determined from the data shown in Figs. (2) and (3) by the method of least squares with the following results

$$A = 1{\cdot}25, \qquad p = 0{\cdot}199, \qquad C = 72, \qquad r = 0{\cdot}867.$$

### (*b*) *Canadian data*

Marshall, Langille and Palmer (1947) have published a description of some work carried out in Canada the primary purpose of which was to relate the radar response from precipitation to the size distribution of raindrops in the precipitation. The filter paper method was used.

In a subsequent paper Marshall and Palmer (1948) have revised some of the conclusions presented in the earlier paper. They state that, from their measurement of raindrop sizes the following formulae can be obtained

$$W = 72\ I^{0{\cdot}88}\ \text{mm}^3/\text{m}^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$Z = 220\ I^{1{\cdot}60}\ \text{mm}^6/\text{m}^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where $W$, $Z$ and $I$ are as defined for Eqs. (2), (3) and (4) and $I$ is measured in mm/hr.

Marshall and Palmer also state that, except at small diameters, the drop size distribution in rain can be represented by

$$N_x = N_0 \, e^{-bx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where $N_x \, \Delta x$ is the number of drops per unit volume having diameter between $x$ and $x + \Delta x$. With the centimetre as the unit of length

$$N_0 = 0{\cdot}08 \text{ cm}^{-4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$b = 41 \, I^{-0{\cdot}21} \text{ cm}^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

In the paper two small scale graphs are given, one showing the experimental points upon which Eq. (13) is based and the other comparing curves based upon Eq. (13) with experimental distribution curves for the three values 1 mm/hr, 5 mm/hr and 25 mm/hr of $I$. Unfortunately the small scale plot of the experimental data in Marshall and Palmer's paper is scarcely adequate to permit the determination of the original experimental values. Accordingly the following indirect methods were adopted.

If Eqs. (1), (2) and (3) can represent the Canadian data then we see from a comparison of Eqs. (3) and (11) and of Eqs. (6) and (12) that

$$C = 72 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$r = 0{\cdot}88 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$r + 3p = 1{\cdot}60 \text{ and so } p = 0{\cdot}24 \quad . \quad . \quad . \quad . \quad (18)$$

Now from Eq. (13) it is easily shown that the fraction of liquid water in the air composed of drops with diameter exceeding $x$ is given by

$$e^{-bx}\left(\frac{b^3x^3}{6} + \frac{b^2x^2}{2} + bx + 1\right) \quad . \quad . \quad . \quad . \quad (19)$$

where $b$ is given by Eq. (15). In the notation of the present paper expression (19) is represented by $F$ and the value of $p$ is known from Eq. (18). Thus we can evaluate expression (19) for selected values of $I$ and $x$ and treat these computed values as experimental observations for the purpose of determining $n$ and $A$. This has been done. The selected values of $I$ and $x$ were

| | |
|---|---|
| $I = 1$ mm/hr | $x = 1$, 1·5 and 2 mm |
| $I = 5$ mm/hr | $x = 1{\cdot}5$, 2 and 2·5 mm |
| $I = 25$ mm/hr | $x = 2$, 2·5 and 3 mm |

The resulting nine values of $(1 - F)$ were used to determine $A$ and $n$ by the method of least squares, the results being

$$A = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$n = 1{\cdot}849 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

These values of $A$ and $n$ are comparable with those obtained from other data but that does not of itself justify the application of Eqs. (1) and (2) to the Canadian data. However Table IV compares the value of expression (19) (*i.e.* values based on Eqs. (13) and (15) with values based on Eqs. (1) and (2) using values of $p$, $A$ and $n$ from (18), (20) and (21).

TABLE IV. Representation of canadian data by eq. (1)

| $I$ mm/hr | $x$ mm | Value of $F$ From Exp. (19) | From Eq. (1) |
|---|---|---|---|
| 1 | 1 | 0·414 | 0·368 |
| | 1·5 | 0·138 | 0·120 |
| | 2·0 | 0·037 | 0·027 |
| 5 | 1·5 | 0·362 | 0·355 |
| | 2·0 | 0·165 | 0·171 |
| | 2·5 | 0·067 | 0·070 |
| 25 | 2·0 | 0·401 | 0·422 |
| | 2·5 | 0·236 | 0·271 |
| | 3·0 | 0·130 | 0·161 |

Reference to the figure in Marshall and Palmer's paper shows that Eq. (13) overestimates the number of raindrops at the smaller diameters, the experimental curve reaching the line representing Eq. (13) at 1 mm for $I = 1$ mm/hr, 1·45 mm for $I = 5$ mm/hr and 1·7 mm for $I = 25$ mm/hr. Thus it seems reasonable to assume that Eqs. (1), (2) and (3) can adequately represent the Canadian data. A further check is provided by inserting the values of $C$, $A$ and $n$ in Eq. (7). This gives $Z_0 = 199$ compared with the value of 220 in equation 12 as quoted by Marshall and Palmer.

### (*c*) *Hilo data*

Anderson (1948) has described some measurements (by the filter paper method) of drop size distribution in rain near Hilo, Hawaii. These measurements were made in connection with a study of the attenuation of 1·25 cm radiation through rain. Anderson gives his results in the form of a graph showing experimental values of $D_{50}$ plotted against $I$. A mean curve is drawn through the experimental points. Against each plotted point on the figure is also written the value of $(D_{75} - D_{25})/2$. The data provided by Anderson are inadequate to test whether his results can be represented by Eq. (1). If however we assume that this form of equation is applicable it is possible to deduce approximate values for the constants involved as follows.

From each of the experimental points plotted in the figure in Anderson's paper the value of $I$ and of $D_{50}$ was read and the value of $(D_{75} - D_{25})/2$ noted. These sets of values were then grouped according to the value of $I$ and mean values obtained for $I$, $D_{50}$ and $(D_{75} - D_{25})/2$. Then the values of $n$ and of $a$ were obtained using Eqs. (5). The results of these computations are given in Table V.

TABLE V. VALUES OF CONSTANTS FROM HILO DATA

| Range of $I$ | No. of points | Mean $I$ | $D_{50}$ | $\frac{(D_{75}-D_{25})}{2}$ | $n$ | $a$ |
|---|---|---|---|---|---|---|
| mm/hr | | mm/hr | mm | mm | | mm |
| 2 | 1 | 2·0 | 0·95 | 0·11 | 4·9 | 1·024 |
| 3 — 5 | 2 | 3·5 | 1·17 | 0·23 | 3·9 | 1·287 |
| 5 — 10 | 17 | 9·2 | 1·57 | 0·28 | 4·4 | 1·706 |
| 10 — 20 | 16 | 16·2 | 1·87 | 0·29 | 5·1 | 2·01 |
| 20 — 30 | 10 | 25·5 | 1·95 | 0·33 | 4·7 | 2·11 |
| 30 — 50 | 10 | 37·5 | 2·19 | 0·44 | 3·9 | 2·41 |
| | | | | Mean | 4·48 | |

Having obtained $n$ and $a$ for certain values of $I$ we can now obtain $W$ from the equation

$$I = 0{\cdot}0036\ W\ \Sigma\ V\ [F(x_2) - F\ (x_1)] \quad . \quad . \quad . \quad . \quad (22)$$

where the numerical coefficient is appropriate to the units employed, $V$ is the terminal velocity of drops having diameter $(x_1 + x_2)/2$ and the summation is taken from $F = 0$ to $F = 1$. For this purpose the range of $F$ was divided into 20 groups. Since Anderson had used values of the terminal velocities of drops as quoted in *Physics of the Air* by Humphreys these same terminal velocities were used in this part of the work. The results were as follows

| | | | | | | |
|---|---|---|---|---|---|---|
| $I$ mm/hr | 2·0 | 3·5 | 9·2 | 16·2 | 25·5 | 37·5 |
| $W$ mm$^3$/m$^3$ | 151 | 232 | 510 | 815 | 1260 | 1756 |

Having obtained $a$ and $W$ for various values of $I$ the best straight lines were fitted by the method of least squares to the logarithms of these quantities yielding the results

$$A = 0{\cdot}881 \qquad p = 0{\cdot}283 \qquad C = 82 \qquad r = 0{\cdot}838$$

where the symbols have the same significance as in Eqs. (2) and (3). As a check on this part of the work values of $D_{50}$ were read from Anderson's mean curve for various values of $I$. By the method of least squares it was found that these could be represented by

$$D_{50} = 0{\cdot}788\ I^{0{\cdot}3}$$

From Eqs. (2) and (5) we see (using a mean value of 4·48 for $n$) that this corresponds to

$$A = 0{\cdot}857 \qquad p = 0{\cdot}300$$

in reasonably good agreement with the values above. Now from Eqs. (6) and (7) we see that, substituting for $A$, $C$, $r$ and $p$ and putting $n = 4{\cdot}48$

$$\begin{aligned} Z &= 1{\cdot}91\ .\ 82\ .\ 0{\cdot}881^3\ .\ I^{1{\cdot}687}\ \ \Gamma\ (1{\cdot}67) \\ &= 96{\cdot}7\ \ I^{1{\cdot}687} \end{aligned}$$

Wexler (1948) however has stated that Anderson's data yield the result

$$Z = 208\ I^{1{\cdot}53}$$

Wexler does not state whether he had access to Anderson's original data but the inadequacy of the data in Anderson's published paper suggests that this was so. We must conclude then that either Eq. (1), the basis of the present treatment, does not adequately represent Anderson's data or that Wexler and Anderson have interpreted the results differently.

It will be noticed that the mean value obtained for $n$ is very different from that obtained from other data. Anderson states that his measurements were made in orographic rain and it is possible that the results differ from those of other workers on that account.

### (*d*) *Lenard*

Lenard (1904) used the filter paper method to measure raindrop sizes. He does not state the number of drops measured but some of his figures seem rather irregular and it appears likely that the number of measurements was small in comparison with the other data already discussed. Lenard gives his results in the form of a number size distribution of drops for each of 13 separate measurements (two measurements were made in one rainfall and three in another but the other eight measurements were in different rainfalls). For each measurement he also quotes the rate of rainfall. Fortunately the rates of rainfall fall into two well defined groups. In series 1 to 7b (series numbers as in Lenard's paper) the intensity of rainfall varied from 0·08 to 7·8 mm/hr and in the remaining series from 15·6 to 43·2 mm/hr. Accordingly the drop size distributions for series 1 to 7b have been added together and the remaining series (omitting No. 9 which had a very irregular distribution of drop sizes) treated similarly. For each group $F$ has been computed for various values of $x$. The results are plotted in Fig. 1. It can be seen that satisfactory straight lines are again obtained. $W$ was also computed for each group. From Fig. 1 values of $a$ and $n$ were obtained and subsequently values of $A$, $p$, $C$ and $r$ computed. The results of these measurements and computations are given in Table VI, $a$ and $W$ being also plotted in Figs. 2 and 3.

TABLE VI. VALUES OF CONSTANTS FROM LENARD'S OBSERVATIONS

| $I$ mm/hr | $n$ | $a$ mm | $W$ mm$^3$/m$^3$ |
|---|---|---|---|
| 3·24 | 2·18 | 1·95 | 163 |
| 22·8 | 3·00 | 3·31 | 840 |

$A = 1{\cdot}42$, $p = 0.272$, $C = 61$, $r = 0{\cdot}840$

### (*e*) *Niederdorfer* (1932), *Defant* (1905), *and Kelkar* (1945)

These three authors do not state the rates of precipitation associated with their measurements. Each of these three grouped his results according to the nature of the rainfall. From the results given it was possible in each case to construct

a ' cumulative number ' curve and hence to evaluate $F(x)$ for various values of $x$. This information was then used to plot log $\log_{10} 1/(1—F)$ against $\log_{10}x$ in Fig. 1, thus testing the validity of Eq. (1).

*Niederdorfer* (1932). In this paper the rates of rainfall are not quoted. Instead the author divides his results into ' general rains ' (five series) and ' thunder rains ' (three series). Moreover the author does not give the sizes of the largest drops In the general rains series a total of 8,005 drops were measured of which three had unspecified sizes greater than 2·59 mm diameter. In the following paragraphs these three drops have been ignored. In the thunder rains a total of 1,647 drops were measured but the sizes of 95 drops exceeding 2·59 mm diameter are not given. From the text, however, it appears that the biggest drop had a diameter of 5·42 mm. From this information a cumulative number curve was plotted and the size distribution of the 95 large drops assessed. Inevitably this procedure introduces some errors but in effect it is equivalent to ' smoothing ' the upper part of the cumulative number curve and it is probable that the errors are not serious.

*Defant* (1905). The difficulties in using Niederdorfer's data are present in exactly the same form in using Defant's data. Defant divided his results into those obtained from continuous rains (seven series) and those obtained from shower or thunder rains (13 series). The continuous rain samples contained a total of 2,438 drops of which no sizes are given for the 13 drops with diameter exceeding 2·15 mm. These 13 drops have been ignored in the following paragraphs. The thunder rain samples contained a total of 4,049 drops with 260 drops having unspecified diameters exceeding 2·15 mm. A search through the text of the paper found the statement that the largest drop had a weight of 140 mgm ($x = 6·45$ mm) and the sizes of 87 other crops which exceeded 2·15 mm in diameter were mentioned in the text in discussion of the suggestion that raindrop sizes occur in multiples of a basic volume. It was not possible to ascertain the sizes of all the 260 large drops. A cumulative number curve was accordingly constructed as for Niederdorfer's data and a size distribution estimated from that.

*Kelkar* (1945) has published a brief account of the measurement of drop size distribution in rain by means of a method similar to the filter paper method — except that smooth paper was used instead of filter paper. He gives a table showing the average percentage of the precipitation due to drops having diameters falling within certain ranges, the table being subdivided for drizzle, light showers, moderate showers, heavy showers and very heavy showers. Using the terminal velocities quoted above Kelkar's data have been used to plot log $\log_{10} 1/(1—F)$ against $\log_{10}x$ for each category of precipitation except drizzle (for which the number of points was too small).

In Fig. 1 the $F$ function has been plotted for each group from each of these three authors. The resulting plots are reasonably linear so that Eq. (1) is applicable. The deduced values of $a$ and $n$ are given in Table VII.

TABLE VII. VALUES OF CONSTANTS IN EQ. (1) FROM DATA OF THREE AUTHORS

| Author | Precipitation | Number of drops | $a$ mm | $n$ |
|---|---|---|---|---|
| Niederdorfer .. | General rains | 8002 | 1·20 | 2·45 |
| | Thunder rains | 1647 | 3·34 | 2·08 |
| Defant .. | Continuous rains | 2438 | 1·23 | 3·00 |
| | Shower or thunder | 4049 | 2·82 | 2·05 |
| Kelkar .. | Light showers | 3211 | 1·27 | 2·54 |
| | Moderate showers | 2310 | 1·61 | 4·32 |
| | Heavy showers | 366 | 1·96 | 6·10 |
| | Very heavy showers | 2415 | 3·52 | 2·05 |

From Table VII it is clear that there is a substantial variation in $n$ and, as will be seen later, the mean value of $n$ in Table VII (3·07) is in excess of the mean value obtained from other investigators. This is not unexpected since all Kelkar's results are associated with showers. The significant feature of these results is not that they show a variation in $n$ but rather that with such small numbers of drops the plots in Fig. 1 do approximate so well to straight lines.

Without knowledge of the rates of precipitation it is, of course, not possible to evaluate the constants in Eqs. (2) and (3).

### (*f*) *Landsberg and Neuberger* (1938)

In some ways these data are the most interesting examined. 750 frozen raindrops, each consisting of glass-clear ice, were collected and measured through a low-power magnifying lens. The measurements obtained were thus independent of filter paper calibration curves. The $F$ function has been plotted in Fig. 1 excluding 11 drops corresponding to values of $F$ less than 0·7 per cent of the volume of ice in the air. The line in Fig. 1 was computed by the method of least squares, the values of $n$ and $a$ being 3·33 and 2·21 respectively. The data given do not permit computation of the rate of rainfall.

### (*g*) *Bentley* (1904)

This author used the flour method to measure raindrop sizes. He seems to have been largely concerned with assessing size distribution against the part of the storm in which it occurred and it has not been possible to analyse his results by the method adopted in this paper.

## 6. COMPARISON OF RESULTS

The results of the computations described above have been collected together in Table VIII for all cases where it has been possible to evaluate $I$. The values given for $n$ are the mean values from the various sets of experiments. $Z_0$ (from Eq. (7)) and $r + 3p$ have also been computed and are tabulated.

TABLE VIII. VALUES OF PARAMETERS IN DROP SIZE DISTRIBUTION

| | $n$ | $A$ | $p$ | $C$ | $r$ | $Z_0$ | $r + 3p$ |
|---|---|---|---|---|---|---|---|
| Ynyslas .. | 2·49 | 1·18 | 0·203 | 74 | 0·845 | 257 | 1·454 |
| Shoeburyness | 2·29 | 1·56 | 0·209 | 59 | 0·816 | 505 | 1·443 |
| Lenard .. | 2·59 | 1·42 | 0·272 | 61 | 0·840 | 360 | 1·656 |
| Laws, Parsons | 2·29 | 1·25 | 0·199 | 72 | 0·867 | 317 | 1·464 |
| Canada .. | 1·85 | 1·00 | 0·240 | 72 | 0·880 | 199 | 1·600 |
| East Hill .. | 1·99 | 1·38 | 0·269 | 65 | 0·829 | 436 | 1·636 |
| Hilo .. | 4·48 | 0·88 | 0·283 | 82 | 0·838 | 97 | 1·687 |
| | | | Mean values excluding Hilo | | | | |
| | 2·25 | 1·30 | 0·232 | 67 | 0·846 | 346 | 1·542 |

There are a number of points to be noted in connection with the numerical values in this table.

(*a*) These numerical values are appropriate when $x$ is measured in mm, $W$ in $mm^3/m^3$ and $Z$ in $mm^6/m^3$.

(*b*) In order that the table shall be self consistent the value 199 for $C$ from the Canadian data, *i.e.*, the value obtained above, has been preferred to the more accurate value of 220 obtained directly by the authors of that work. The difference however is small.

(*c*) The mean values of $C$, $A$ and $n$ given in Table VIII lead to a value of 335 for $Z_0$ if substituted in Eq. (7). Naturally this is very close to the average of the values for $Z_0$ obtained from each set of data separately.

(*d*) The validity of Table VIII is of course dependent upon the applicability of Eqs. (1), (2) and (3). Of these perhaps Eq. (1) is the most important. The evidence in support of the form of Eq. (1) comprises the linearity of the lines in Fig. 1 and the agreement between the two ways of computing $F$ as shown in Table IV.

(*e*) The values in Table VIII are appropriate only in the region where the terminal velocities of the drops are as quoted in Table I, *i.e.*, near the surface. At greater heights the terminal velocities will be smaller and the values of $W$ must be increased accordingly.

(*f*) The Hilo values have been excluded from the means because the values of $n$ and $A$ differ significantly from the other values and the author of that work states that the observations were made in a special type of precipitation, *viz.*, orographic.

Eqs. (1), (2) and (3) should give a reasonably close representation of the average distribution of liquid water in the air near the ground during precipitation, if the mean values of the parameters in Table VIII are inserted in the equations, for about 80 per cent of the water, *i.e.*, excluding the 20 per cent of the water comprised by the smallest and the largest drops. It is probable however that

the mean values of the parameters will not be appropriate either in purely orographic rain or in rain which is essentially of the showery type.

## 7. Numerical application of the formulae

Using the mean values of $n$, $A$, $p$, $C$ and $r$ given at the foot of Table VIII the distribution of liquid water and of number of drops according to drop diameter have been computed from Eqs. (1), (2) and (3) for various rates of rainfall.

TABLE IX. Values of $a$ and of $W$

| $I$ mm/hr | $a$ mm | $W$ $mm^3/m^3$ |
|---|---|---|
| 0·5 | 1·107 | 37 |
| 1·0 | 1·300 | 67 |
| 2·5 | 1·608 | 145 |
| 5·0 | 1·888 | 261 |
| 10·0 | 2·218 | 470 |
| 25·0 | 2·743 | 1020 |

The values of $a$ and $W$ were also computed. The results of these computations are given in Tables IX and X. The tabulated values are given to a higher degree of accuracy than is justified by the experimental data upon which the values of the parameters are based in order to illustrate the variations predicted by the formulae. For completeness the amount of liquid water and the number of drops comprised by drops with diameter less than 0·25 mm have also been tabulated, though it is not suggested that the formulae are applicable to such small drops.

## 8. Deviations from the formulae

In order to assess the closeness with which the formulae fit the observational data the following procedure was adopted.

The value 2·25 obtained above for $n$ is the mean of the average values obtained from each set of data. Each set of data however provided several experimental values of $n$, these values being tabulated in sections 4 and 5. Accordingly the root mean square value of $(n - 2{\cdot}25)$ was computed using the individual values upon which the mean value 2·25 is based. Since the method of treatment of the Canadian data yielded only the average value of $n$, *viz.* 1·85, this value was included five times in order to give weight to that data similar to that given to the other observations. The standard deviation of $n$ computed in this way was 0·41.

A different procedure was adopted with regard to the parameter $a$. This quantity varies with $I$, the rate of precipitation, and the rate of precipitation has been tabulated above for each observed value of $a$. Values of $a$ were computed from Eq. (2) using $A = 1{\cdot}30$ and $p = 0{\cdot}232$ to correspond to each experimental value. Denoting the computed value of $a$ by $a_f$ the root mean square of the quantity $(a_f - a)/a_f$ was computed. Again special steps were necessary to give adequate weight to the Canadian data. Five 'observational' values of $a$ were computed for the Canadian data using Eq. (2) and the values 1·0 and 0·24 for $A$ and $p$,

TABLE X. DISTRIBUTION OF LIQUID WATER AND OF NUMBER OF DROPS

| $I$ mm/hr | 0·50 | 1·00 | 2·50 | 5·00 | 10·0 | 25·0 | 0·50 | 1·00 | 2·50 | 5·00 | 10·0 | 25·0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Drop Diameter mm | Liquid water $mm^3/m^3$ | | | | | | Number of drops per $m^3$ | | | | | |
| 0·00 — 0·25 | 1·3 | 1·6 | 2·2 | 2·7 | 3·4 | 4·6 | 1250 | 1585 | 2136 | 2685 | 3369 | 4538 |
| 0·25 — 0·50 | 4·4 | 5·7 | 7·9 | 10·1 | 12·7 | 17·3 | 160 | 208 | 287 | 364 | 461 | 625 |
| 0·50 — 0·75 | 6·9 | 9·5 | 13·8 | 17·9 | 23·1 | 31·8 | 54 | 74 | 108 | 140 | 180 | 248 |
| 0·75 — 1·00 | 7·7 | 11·6 | 18·3 | 24·8 | 32·9 | 46·4 | 22 | 33 | 52 | 71 | 94 | 132 |
| 1·00 — 1·25 | 6·8 | 11·7 | 20·6 | 29·7 | 41·0 | 59·9 | 9 | 16 | 28 | 40 | 55 | 80 |
| 1·25 — 1·50 | 4·8 | 10·0 | 20·6 | 31·9 | 46·5 | 71·3 | 3·55 | 7 | 15 | 23 | 34 | 52 |
| 1·50 — 1·75 | 2·9 | 7·3 | 18·4 | 31·5 | 49·0 | 79·8 | 1·27 | 3·27 | 8 | 14 | 22 | 35 |
| 1·75 — 2·00 | 1·4 | 4·7 | 14·9 | 28·7 | 48·6 | 84·9 | 0·41 | 1·37 | 4·33 | 8 | 14 | 25 |
| 2·00 — 2·25 | 0·6 | 2·6 | 11·1 | 24·5 | 45·5 | 86·5 | 0·11 | 0·53 | 2·20 | 4·87 | 9 | 17 |
| 2·25 — 2·50 | 0·2 | 1·3 | 7·5 | 19·4 | 41·4 | 84·6 | 0·03 | 0·19 | 1·07 | 2·76 | 6 | 12 |
| 2·50 — 2·75 | 0·1 | 0·6 | 4·6 | 14·4 | 34·1 | 79·9 | 0·01 | 0·06 | 0·49 | 1·52 | 3·60 | 8 |
| 2·75 — 3·00 | | 0·2 | 2·6 | 10·1 | 27·5 | 73·0 | | 0·02 | 0·21 | 0·81 | 2·21 | 6 |
| 3·00 — 3·25 | | 0·1 | 1·4 | 6·6 | 21·1 | 64·3 | | 0·004 | 0·09 | 0·41 | 1·32 | 4·03 |
| 3·25 — 3·50 | | | 0·7 | 4·0 | 15·4 | 55·0 | | | 0·03 | 0·20 | 0·77 | 2·74 |
| 3·50 — 3·75 | | | 0·3 | 2·4 | 10·8 | 45·6 | | | 0·01 | 0·10 | 0·43 | 1·83 |
| 3·75 — 4·00 | | | 0·1 | 1·3 | 7·2 | 36·6 | | | 0·004 | 0·04 | 0·24 | 1·20 |
| 4·00 — 4·25 | | | | 0·6 | 4·6 | 28·5 | | | | 0·02 | 0·13 | 0·77 |
| 4·25 — 4·50 | | | | 0·3 | 2·8 | 21·6 | | | | 0·01 | 0·06 | 0·49 |
| 4·50 — 4·75 | | | | 0·1 | 1·6 | 15·8 | | | | 0·003 | 0·03 | 0·31 |
| 4·75 — 5·00 | | | | 0·1 | 0·9 | 11·2 | | | | 0·001 | 0·015 | 0·19 |
| 5·00 — 5·25 | | | | | 0·5 | 7·8 | | | | | 0·007 | 0·11 |
| 5·25 — 5·50 | | | | | 0·2 | 5·2 | | | | | 0·003 | 0·06 |
| 5·50 — 5·75 | | | | | 0·1 | 3·4 | | | | | 0·001 | 0·04 |
| 5·75 — 6·00 | | | | | 0·1 | 2·1 | | | | | 0·001 | 0·02 |
| 6·00 — 6·25 | | | | | | 1·3 | | | | | | 0·011 |
| 6·25 — 6·50 | | | | | | 0·8 | | | | | | 0·006 |
| 6·50 — 6·75 | | | | | | 0·4 | | | | | | 0·003 |
| 6·75 — 7·00 | | | | | | 0·3 | | | | | | 0·001 |
| 7·00 — 7·25 | | | | | | 0·1 | | | | | | |

these being the values of these parameters computed solely from these data, the observational values being computed for values 1, 5, 10, 15 and 25 mm/hr of $I$. The percentage standard deviation of $a$ computed in this way from all the data contributing to the mean values at the foot of Table VIII was 18·3 per cent.

The procedure adopted with regard to the standard deviation of $W$ was similar to that for $a$ and the result was 13·5 per cent.

These standard deviations do not represent the standard deviations of single measurements of drop size distribution from that predicted by Eqs. (1), (2) and (3). These would depend on the size of the filter paper used, the duration of exposure and various other factors. The figures quoted above can be used only to indicate the closeness with which the experimental data, treated in the way described above, fit the formulae deduced.

## 9. Effect of variation in the numerical values of the parameters

If we regard the parameter $a$ as determining the scale upon which the drop diameters are measured we see that the drop size distribution is the same for all rates of rainfall. This is of assistance in appreciating the effect of variations in $a$ upon the drop size distribution. If we have two distributions in which the parameter $a$ has values in the ratio $k$ to 1 the fraction of liquid water comprised by drops with diameter between $x_1$ and $x_2$ in the distribution with parameter $a$ will be comprised by drops with diameter between $kx_1$ and $kx_2$ in the distribution with parameter $ka$. The ratio of the number of drops between $x_1$ and $x_2$ in the '$a$' distribution to the number of drops between $kx_1$ and $kx_2$ in the $ka$ distribution will be $k^3$.

The effect of variations in $W$ is self evident.

It is clear that an increase in the value of $n$ will decrease the spread of the distribution. In order to illustrate the magnitude of this effect the drop size distribution for $I = 5$ mm/hr has been recomputed using the same values of $a$ and $W$ but with $n = 4$. The results are given in Table XI and may be compared with the corresponding columns of Table X.

TABLE XI. Distribution of liquid water and of drops for $I = 5$ mm/hr, $a = 1{\cdot}888$ mm, $W = 261$ mm$^3$/m$^3$ and $n = 4$

| Drop Diameter mm | Liquid water mm$^3$/m$^3$ | No. of drops |
|---|---|---|
| 0·00 — 0·25 | 0·08 | 79 |
| 0·25 — 0·50 | 1·2 | 43 |
| 0·50 — 0·75 | 5·1 | 40 |
| 0·75 — 1·00 | 13·3 | 38 |
| 1·00 — 1·25 | 25·9 | 35 |
| 1·25 — 1·50 | 40·2 | 30 |
| 1·50 — 1·75 | 50·5 | 22 |
| 1·75 — 2·00 | 50·8 | 15 |
| 2·00 — 2·25 | 39·5 | 7·9 |
| 2·25 — 2·50 | 22·7 | 3·2 |
| 2·50 — 2·75 | 9·2 | 0·97 |
| 2·75 — 3·00 | 2·5 | 0·20 |
| 3·00 — 3·25 | 0·41 | 0·03 |
| 3·25 — 3·50 | 0·04 | 0·002 |

The effect of increasing $n$ from 2·25 to 4·0 has been to transfer the diameter at which the maximum concentration of liquid water occurs from between 1·25 and 1.50 mm to between 1·75 and 2·00 mm and to increase the maximum concentration from 32 $mm^3/m^3$ to 50·8 $mm^3/m^3$ for 0·25 mm range of diameter. Actually if the predominant diameters are computed from Eq. (8) we find that they are 1·454 and 1·757 mm in the two cases. To a large extent the effect of the change in $n$ is reduced if we consider a larger range of diameters embracing the predominant diameter. Thus if we consider the drops having diameters between 0·75 mm and 2·50 mm we find that the amounts of liquid water are 191 $mm^3/m^3$ and 243 $mm^3/m^3$ for $n = 2·25$ and 4·0 respectively and the effect of the variation of $n$ on the number of drops is even less marked, the numbers being 163·6 and 151·1/$m^3$ for $n = 2·25$ and $n = 4·0$ respectively.

## 10. Acknowledgments

The ready co-operation of L. G. Hemens, R. F. Jones and A. R. Laird in providing the data described in section 4 is gratefully acknowledged. Thanks are also due to J. K. Bannon, B.A., for the assistance which always results from the discussion of difficulties. Finally acknowledgment is made to the Director, Meteorological Office, with whose permission this paper is published.

## References


| | | |
|---|---|---|
| Anderson, L. J. | 1948 | *Bull, Amer. Met. Soc.*, Boston, Mass, **29**, p. 362. |
| Bentley W. A. | 1904 | *Mon. Weath. Rev.*, Washington, D.C., **32**, p. 450. |
| Defant, A. | 1905 | *Wiener Sitzungsber.* Math.— Naturwiss. Klasse, Wien, **114**, p. 585. |
| Kelkar, V. N. | 1945 | *Proc. Ind. Acad. Sci.*, **22**, p. 394. |
| Landsberg, H. and Neuberger, H. | 1938 | *Bull. Amer. Met. Soc.*, Boston, Mass., **19**, p. 354. |
| Laws, J. O. and Parsons, D. A. | 1943 | *Trans. Amer. Geoph. Union*, Washington, p. 452. |
| Lenard, P. | 1904 | *Met. Z.*, Braunschweig, **21**, p. 249. |
| Marshall, J. S., Langille, R. C. and Palmer, W. McK. | 1947 | *J. Met. Amer. Met. Soc.*, Boston, Mass., **4**, p. 186. |
| Marshall, J. S. and Palmer, W. McK. | 1948 | *Ibid.*, **5**, p. 165. |
| Niederdorfer, E. | 1932 | *Met. Z.*, Braunschweig, **49**, p. 1. |
| Wexler, R. | 1948 | *J. Met. Amer. Met. Soc.*, Boston, Mass., **5**, p. 171. |

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# Notes on the measurement and estimation of atmospheric radiation — 2

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SUMMARY

An account is given of further measurements of atmospheric radiation which confirm former conclusions regarding the errors of two types of radiation chart when used to compute total radiation at the surface. The greater part of the difference between computed and observed radiation is attributed to two causes — emission from particulate matter and the variation of emissivity of water vapour with temperature. The results suggest a form for this variation between —5°C and +25°C.

Some measurements and computation of the change of radiative flux in the lowest metre at night, *i.e.*, of the direct radiative cooling of the air, are reported. Miscellaneous subjects discussed include the radiation from atmospheric dust, the radiative temperature of a short grass surface and its measurement by simple means in all conditions, and an anomalous occasion with very high incoming radiation for which no explanation could be found. Some comments are offered on the experimental justification of two possible uses of the radiation charts.

## 1. INTRODUCTION

The work here reported extends and amplifies that carried out at Kew Observatory and described by the author in a previous paper (Robinson 1947) which will be referred to as Part I. Familiarity with the methods set out and terms introduced in Part I will be assumed ; the same techniques of observation and reduction of results were used throughout. As a result of the previous work a method of constructing an empirical radiation chart was suggested. This chart has been much used in the present discussion, and for convenience it will be termed the Kew chart. It is illustrated in Fig. 1, some of the grid lines having been omitted for clarity. In its construction the method of F. A. Brooks (Part I, paragraphs 10 and 11) was used to obtain the emissivity of various amounts of water vapour, and it was found that five of the sets of observations formed a homogeneous group giving a well defined curve of emissivity against radiating path length, whilst all the other observations gave various higher emissivities. The chart was based only on the five homogeneous sets of observations which were termed cases of " minimum radiation," and the variable " additional radiation " present in the other observations was attributed to radiators other than water vapour and carbon dioxide. The observations now reported contain five further " minimum radiation " cases, and so double the data on which the chart is based.

The early part of 1947, during which most of the observations now reported were taken, brought extremes both of heat and cold which are rarely reached in Southern England, and as a result the investigation has been extended over a range of values of total atmospheric radiation at the surface which includes all those of practical interest in this locality.

## 2. STANDARDISATION OF THE RADIOMETER

The Linke-Feussner radiometer has been standardised at intervals over a period of more than two years, and the results are shown in Table I. Since the sensitivity of the galvanometer and resistance of the leads were changed from time

to time, the sensitivity is expressed as the thermopile output in microvolts produced by exposure of the instrument to a net radiative intensity of 1 milliwatt per sq. cm. (N.B. This is not the absolute sensitivity of the thermopile in $\mu$v/mw). There

TABLE I—Calibration of the Linke-Fuessner radiometer

| Date | Air temperature °C | Vapour Pressure mb | Sensitivity $\mu$v mw$^{-1}$ cm$^2$ |
|---|---|---|---|
| 11 September 1946 | 16 | 20 | 1·111 |
| 10 October 1946 | 14 | 11 | 1·121 |
| 15 October 1946 | 18 | 10 | 1·102 |
| 20 November 1946 | 19 | 13·5 | 1·126 |
| 30 June 1947 | 18 | 15 | 1·112 |
| 11 September 1947 | 19 | 15 | 1·129 |
| 3 November 1947 | 17 | 13 | 1·134 |
| 16 April 1948 | 13 | 7·5 | 1·137 |
| 7 June 1948 | 15 | 10·5 | 1·128 |
| 27 October 1948 | 6 | 6 | 1·138 |

appears to be significant evidence of a small discontinuous change in pile sensitivity between June and September 1947, for which no reason is known. The mean sensitivities are :

September 1946 — July 1947 $1{\cdot}115 \pm 0{\cdot}007$ $\mu$v mw$^{-1}$cm$^2$

September 1947 — October 1948 $1{\cdot}133 \pm 0{\cdot}004$ $\mu$v mw$^{-1}$cm$^2$ *

In view of arguments to be developed in section 5 it is important to consider the question of temperature variation of sensitivity of the instrument, briefly men-

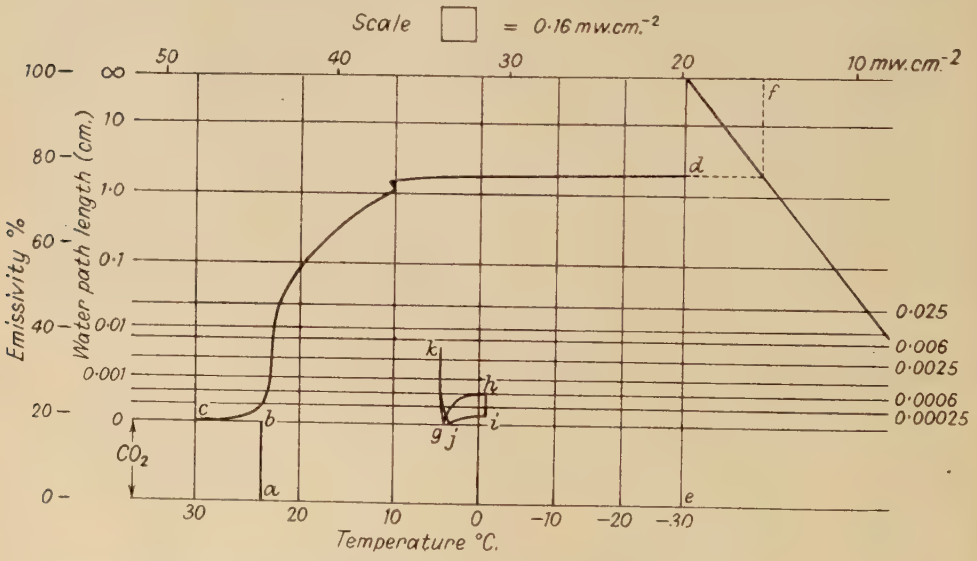


Figure 1. The Kew radiation chart with examples of its use.

(a) Downward flux of radiation, at surface 28 May 1947, 1300 G.M.T. is given by area *abcdea* + reading *f*.

(b) Change in net (upward) radiative flux between 20 cm and 120 cm, 24 November 1948, 1800 G.M.T. is given by area *ijkghi* (" clockwise areas " positive).

* Mean deviations are given here and elsewhere in the paper.

tioned in Part I, section 6. The temperature coefficient of the constantan manganin junctions over the range 0°C to 30°C is approximately + 0·2 per cent/°C. About two-thirds of the total circuit resistance is of copper, and the remainder of resistance alloys ; the temperature coefficient of circuit resistance is therefore rather less than + 0·3 per cent/°C, and, in calibration conditions, with the pile and the galvanometer at the same temperature, the temperature coefficient of sensitivity of the whole instrument would be expected to be rather less than —0·1 per cent/°C. The second set of calibrations (September 1947—October 1948) is consistent with such a value, for although the individual determinations are not sufficient in number or precision to give a sound value of the coefficient, the decrease of sensitivity with increase of temperature seems to be just significant. In normal use, as distinct from calibration, the radiometer and galvanometer were at different temperatures, and almost all the varying (copper) resistance is incorporated in the galvanometer. No check of galvanometer temperatures was kept, but in general they followed the trend of thermopile temperatures with extreme limits of about + 5°C and + 20°C corresponding to radiometer temperatures of —2°C to +30°C. Hence it was not possible to apply a temperature correction to the observed radiation, but it can be confidently assumed that the variation of sensitivity was within the limits ±0·1 per cent per degree C change of radiometer temperature. The possibility that a variation with wave length of the emissivity of the pile blackening might affect the results was checked by using a sufficiently wide range of black body temperature during calibration. No perceptible departure from Stefan's law was found for source temperatures from —30°C to +50°C and a pile temperature about 20°C. These conditions cover the whole range of the observations. I am indebted to Dr. K. H. Stewart and Mr. B. A. Eagle for this calibration.

In reducing each observation the sensitivity determined at the previous calibration was used (only one observation lies in the doubtful period of July — September 1947), so that systematic errors of about ± 0·5 per cent may have been introduced. The error of an individual determination was estimated (Part I) to be about ± 1·5 per cent. The relative accuracy of the measured intensities at various zenith angles during an individual run should be better than this, and was probably only limited by the smallest detectable change of radiation, which was about 0·05 mw $cm^{-2}$, corresponding to a galvanometer deflection of 0·1 mm.

### 3. Comparison of measured and computed values of the total radiation

The Linke-Fuessner instrument was used exclusively in the observations to be discussed, but the method of taking readings on seven zones of the sky, at zenith angles of 0°, 22½°, 37½°, 52½°, 67½° 74° and 80°, was adopted, in preference to the single reading at 52½°. The equality of the intensity at 52½° to the total flux was confirmed. The computation of the radiation was carried out on all occasions using the upper air temperature and humidity data from the routine Larkhill and Downham Market radio-sonde ascents. Table II shows the results of measurement and computation, using both the Elsasser and Kew charts, for all occasions for which reliable upper air data could be obtained, *i.e.*, excluding cases when

there was either a temperature difference of more than 2°F at corresponding lower levels between Downham Market and Larkhill, or a deep surface inversion. The table contains only those cases for which visibility was better than 1,000 yd and no cloud was observed, as in Table V of Part I. "Minimum radiation" cases are marked with an asterisk. Details of the computation for 28 May 1947, 1300 G.M.T. are shown in Fig. 1.

TABLE II—Observed and computed atmospheric radiation

| Date and time G.M.T. | | | Screen Temp °C | Reduced path length cm water | Observed radiative flux mw cm$^{-2}$ | Computed radiative flux mw cm$^{-2}$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | Kew Chart | Elsasser Chart |
| * 1 March | 47 | 0100 | —2 | 0·40 | 19·6 | 20·4 | 22·2 |
| 7 March | 47 | 1800 | 1 | 0·41 | 20·9 | 20·9 | 23·3 |
| 9 March | 47 | 1200 | 7 | 0·28 | 23·8 | 24·1 | 25·3 |
| *15 March | 47 | 0030 | 0 | 0·24 | 20·2 | 21·2 | 23·5 |
| 21 March | 47 | 2230 | 8 | 0·66 | 26·0 | 26·1 | 27·8 |
| *25 March | 47 | 0001 | 2 | 0·31 | 22·0 | 22·9 | 25·0 |
| 25 March | 47 | 0900 | 6 | 0·32 | 23·7 | 23·5 | 25·8 |
| 10 April | 47 | 0030 | 3 | 0·38 | 23·2 | 23·7 | 25·6 |
| 10 April | 47 | 1100 | 9 | 0·76 | 26·4 | 27·1 | 28·3 |
| 10 April | 47 | 2300 | 5 | 0·64 | 25·8 | 25·5 | 27·7 |
| 11 April | 47 | 2230 | 7 | 0·56 | 25·7 | 25·6 | 27·2 |
| *13 April | 47 | 2300 | 9 | 0·58 | 25·0 | 26·5 | 28·9 |
| 15 April | 47 | 2330 | 9 | 0·73 | 27·3 | 27·4 | 29·0 |
| 17 April | 47 | 1230 | 17 | 0·65 | 30·5 | 28·9 | 30·4 |
| *28 May | 47 | 1300 | 24 | 1·12 | 32·8 | 33·0 | 34·4 |
| 28 May | 47 | 2200 | 16 | 1·19 | 31·0 | 31·3 | 32·8 |
| 29 May | 47 | 1500 | 31 | 1·68 | 39·1 | 39·6 | 38·4 |
| 10 June | 47 | 0900 | 17 | 1·01 | 30·1 | 29·9 | 31·5 |
| 27 June | 47 | 2230 | 17 | 1·78 | 34·2 | 34·3 | 34·2 |
| 7 August | 47 | 2000 | 15 | 1·15 | 30·1 | 30·4 | 31·9 |
| 17 September | 47 | 0030 | 11 | 1·05 | 28·7 | 28·5 | 30·3 |
| 24 September | 47 | 0930 | 12 | 0·61 | 27·4 | 27·2 | 28·7 |
| 24 September | 47 | 2300 | 8 | 0·59 | 26·5 | 26·1 | 27·7 |
| 1 October | 47 | 1630 | 16 | 0·79 | 30·2 | 29·6 | 30·9 |

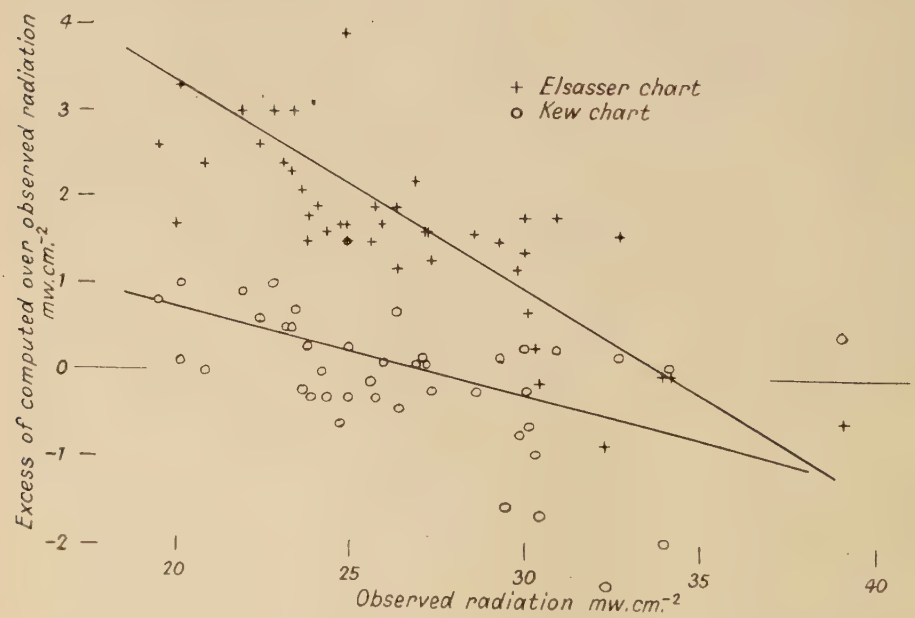


Figure 2. Comparison of computed and observed downward radiation.

Fig. 2 shows for both charts the excess of observed over computed radiation, plotted against observed radiation, and includes the present results and those of Part I. It gives an indication of the errors likely when using the charts to estimate the total atmospheric radiation at the surface. The decrease of error with increase of observed radiation is much more marked with the Elsasser chart, but is significant with the Kew chart. For work in which it is not considered necessary to take account of this effect the mean error of the chart values is of interest. Taking all the observations of Fig. 1 we find

Kew chart — observed = 0·04 ± 0·55 mw/cm$^2$
Elsasser chart — observed = 1·58 ± 0·45 mw/cm$^2$

The new observations contain five cases of " minimum radiation " (Part I, section 10), and for these and the five previous cases we find

Kew chart — observed = 0·5 ± 0·4 mw/cm$^2$
Elsasser chart — observed = 1·8 ± 0·8 mw/cm$^2$

## 4. Determination of atmospheric emissivity

The method of F. A. Brooks (Part I, sections 10 and 11) was used to obtain an empirical curve of emissivity against water vapour path length, and the results are shown in Figs. 3 and 4, corresponding to Figs. 9 and 11 of Part I, the curves being the same in both papers. It will be seen that there is some indication that the curves, which were used in construction of the Kew chart, slightly overestimate the emissivities for path lengths between 0·2 cm (the lower limit of the observations) and about 4 cm, the error suggested being about 1·5 per cent over the greater part of this range. The laboratory experiments of Elsasser and Brooks determine the emissivities quite definitely between 0·0001 and 0·005 cm, but Elsasser's outdoor

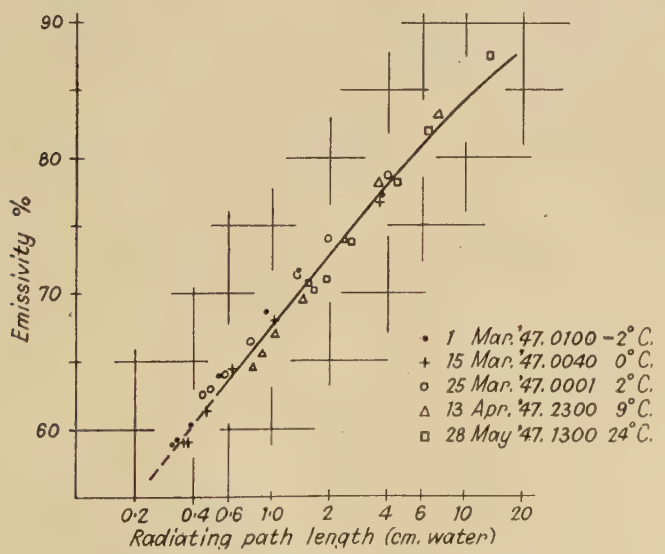


Figure 3. Atmospheric emissivity, E ($u$ & $CO_2$), in minimum radiation conditions.

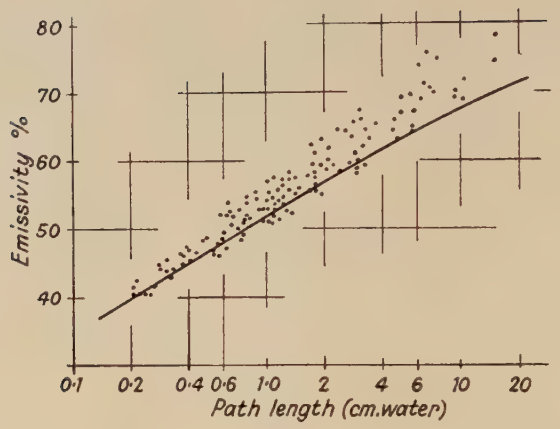


Figure 4. Emissivity of water vapour in the presence of $CO_2$, $E$ ($u$). All observations.

experiments, which bridge the gap between 0·005 and 0·5 cm are not sufficiently consistent to define the emissivity curve more closely than within 2 to 3 per cent. The agreement between the Kew chart and the observed radiation, particularly in the " minimum radiation " cases, can be improved to a small extent by decreasing the emissivities over the range 0·005 to 5 cm to the values shown in Table III,

TABLE III—Isothermal emissivity E ($u$ + $CO_2$) of atmospheric slabs of water path $u$

| $u$ (cm) | E ($u$ + $CO_2$) | $u$ (cm) | E ($u$ + $CO_2$) |
|---|---|---|---|
| 0 | 0·185 | 0·06 | 0·527 |
| 0·0001 | 0·208 | 0·1 | 0·560 |
| 0·00025 | 0·225 | 0·25 | 0·620 |
| 0·0006 | 0·265 | 0·6 | 0·685 |
| 0·001 | 0·290 | 1·0 | 0·725 |
| 0·0025 | 0·335 | 2·5 | 0·790 |
| 0·006 | 0·385 | 6·0 | 0·855 |
| 0·01 | 0·410 | 10·0 | 0·888 |
| 0·025 | 0·463 | 25·0 | 0·940 |

which replaces Table VI of Part I. The computation using these values was made in three examples only, but the effect of the changes can be judged from these. The results are shown in Table IV. An examination of these results in conjunction

TABLE IV—Examples of the revised computation of atmospheric radiation

| Date | | Observed Radiation (mw cm$^{-2}$) | Computed Radiation (mw cm$^{-2}$) Original | Revised |
|---|---|---|---|---|
| 5 September | 46 | 29·4 | 29·6 | 29·4 |
| 1 March | 47 | 19·6 | 20·4 | 20·2 |
| 28 May | 47 | 32·8 | 33·0 | 32·7 |

with Fig. 12 of Part I makes it clear that the Kew chart cannot be further improved merely by an adjustment of emissivities within the limits allowed by the various sets of experimental data. The most obvious explanation of the residual errors lies in the treatment of the temperature variation of emissivity.

### 5. The variation of emissivity with temperature

It has been shown in the previous paragraph that the systematic variation of chart error with observed radiation is probably a temperature effect, and it was found possible to use the new results to investigate this question, which is necessarily neglected in the construction of the Kew chart. For this purpose only the ten minimum radiation cases were considered, and of these the observation of 13 April 1947 was rejected since it gave results inconsistent with the other nine, probably because of the use of incorrect upper air humidities. In Fig. 5 the percentage

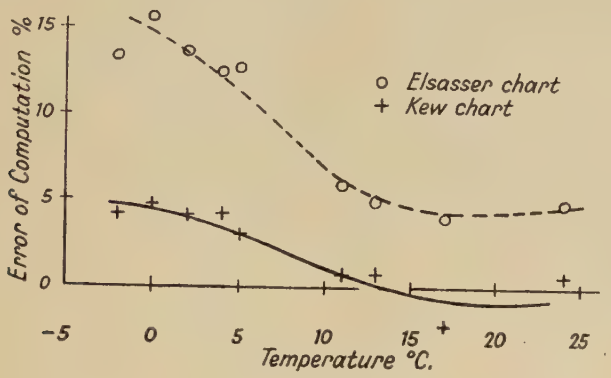


Figure 5. Variation of chart error with temperature.

error for both charts has been plotted against screen temperature, using the data of Table II and of Table V, Part I, the use of screen temperature being justified by the fact that in all observations about 90 per cent of the radiation originated in air layers at temperatures within $\pm$ 3°C of this value. The error shows a systematic variation with temperature for both charts. Now Elsasser's chart allows for a theoretically determined variation of emissivity with temperature ; this variation is shown in the tabulations of his function Q (Elsasser 1942, p. 77). The variation differs for different path lengths, and in Fig. 7 it is plotted for path lengths of 0·25 and 1·0 cm, the values at 20°C being taken as an arbitrary standard. It is possible approximately to remove the effect of this temperature variation from the Elsasser chart computations. For this purpose the atmospheric radiation was considered to arise from an isothermal slab at screen temperature of thickness equal to the zenith value of the reduced path length, as defined in Part I, section 11, and the correcting factor determined by interpolation in a diagram similar to Fig. 7. Using a standard temperature of 20°C a correction of this type was applied to the Elsasser chart values of Fig. 5, and the results are shown in Fig. 6. In this diagram the percentage errors for the Kew chart are again plotted, after applying a small correction to make them consistent with the emissivities of Table III. The two sets of points in Fig. 6 therefore show the errors of radiation computations made with the Kew type of chart, *i.e.*, neglecting the temperature variation of emissivity, basing the

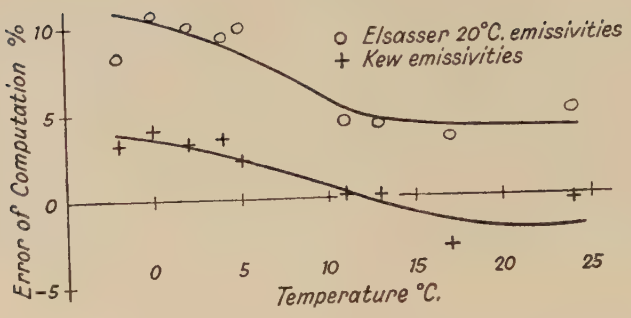


Figure 6. Variation of error of computation with temperature (constant emissivity assumed).

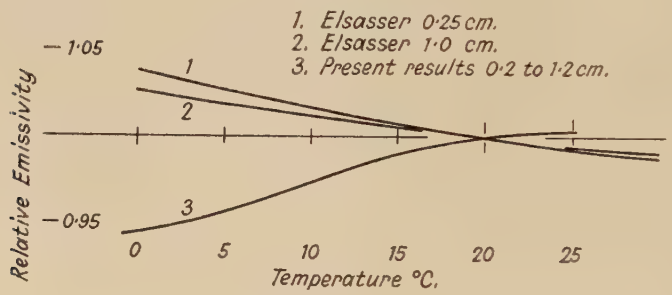


Figure 7. Variation of emissivity of water vapour with temperature.

chart on Table III and on Elsasser's 20°C emissivities respectively. The approximate parallelism of the lines strongly suggests that the errors are due to an excess of about 5 per cent in the Elsasser emissivities coupled with a neglected temperature variation of emissivity of the form shown by curve 3 of Fig. 7, the inverse of the "Kew" curve of Fig. 6. This curve must be regarded as a mean value for the range of reduced path lengths covered by the nine observations on which it is based, *i.e.*, from 0·2 to 1·2 cm. It will be seen that the indicated variation is of the opposite sign to that computed by Elsasser, an increase rather than a decrease of emissivity with increasing temperature. The approximate, empirical, and indeed rather dubious nature of these arguments should not be forgotten. They use methods only justifiable if emissivity is constant with temperature to show that in fact it probably varies, and to indicate the nature of the variation ; but it can be said that they strongly suggest an appreciable increase of emissivity with temperature in the range of path lengths and temperatures considered. It is of some importance to have reliable information on this question, particularly for the shorter path lengths, of the order of 0·001 cm, where Elsasser's computations suggest a very considerable decrease over the range 0 to 40°C. The results of Brooks and of Falckenburg (quoted by Elsasser 1942, p. 84) indicate a notable increase of emissivity at these path lengths between 15 and 30°C, but it would

be very desirable to have this confirmed by a homogeneous series of measurements in which a high degree of relative, if not of absolute, accuracy should be obtainable.

Cowling (1943) has discussed the factors controlling the temperature variation of emissivity. The effect is complex, important contributions being due to the change in width of individual lines, the change in relative intensities of individual lines in the same band, and the shift in the maximum of the black body radiation relative to the spectrum. Because of this complexity it would not be surprising if the computations of Elsasser were in error, particularly in view of the approximate nature of his data for the $6\mu$ band.

### 6. Radiative heating and cooling of the air near the ground

In an unpublished paper (Robinson 1948) the author has discussed the use of the radiation chart in determining the role of radiation in the transfer of heat between the ground surface and the air, and in the temperature changes in the lowest layers. The computations showed that direct radiative heating and cooling of the air was often of great importance, and might in suitable conditions account for a heat transfer 10 to 100 times that necessary to cause the observed temperature changes. In view of the rather unexpected nature of these results it seemed important to attempt to check them by observation. The indicated changes in radiative flux over distances of about one metre, were of the order of 0·1 mw $cm^{-2}$ on nights with a well-developed inversion, and up to about five times this on a hot afternoon with strong insolation. We have seen that the limit of resolution of the instrument is 0·05 mw $cm^{-2}$, so that the daytime conditions seemed to offer the best chance of success. It was found however that fluctuations of the short wave radiation, and probably also of the radiative temperature of the surface, were so large that consistent readings could not be obtained. The time taken over an individual determination of net flux (involving eight shutter movements and galvanometer readings) was so great that there was no hope of smoothing out the fluctuations by taking many observations. Successful results were however obtained in night conditions, the decreased magnitude of the change in flux being more than offset by the greatly increased speed of measurement and steadiness of conditions. The procedure was to have two platforms, one of which supported the thermopile 20 cm from the surface, the other at 120 cm. The instrument was directed on the ground (zenith angle $127\frac{1}{2}°$), and the galvanometer deflection read as the radiometer was placed first on one platform then on the other. Five readings were taken at each height, the instrument was then turned on the sky (zenith angle $52\frac{1}{2}°$) and ten readings taken at each height. Finally five more pairs of readings were taken at zenith angle $127\frac{1}{2}°$. Shutter readings to check stability of zero were obtained after every second or third pair of observations. The surface temperature was taken by means of a grass thermometer, and wet and dry bulb readings at three heights by means of an Assmann psychrometer (on one occasion at eight heights by means of an apparatus in use by Mr. N. E. Rider and the author in another investigation). The measured values of flux change are shown in Table V together with computations made using the Kew chart, an example being shown in Fig. 1.

The relatively large value of the mean deviations shown is in part due to the limit of resolution being 0·05 mw $cm^{-2}$, and in part to fluctuations of surface temperature. The results are considered to confirm the radiation chart predictions, and it is not thought that greater precision can be obtained with the present rather clumsy method. It should be noted that no allowance for $CO_2$ radiation is made in these

TABLE V—Observed and computed values of the change in net radiative flux between 20 cm and 120 cm

| Date | Surface temperature (°A) | 120 cm temperature (°A) | Change of radiative flux (mw $cm^{-2}$) Observed | Computed |
|---|---|---|---|---|
| 1 November 48 | 271·2 | 277·5 | 0·12 ± 0·16 | 0·16 |
| 24 November 48 | 271·2 | 277·2 | 0·07 ± 0·13 | 0·15 |
| 25 November 48 | 271·5 | 277·2 | 0·19 ± 0·08 | 0·13 |
| 27 January 49 | 272·5 | 279·0 | 0·28 ± 0·23 | 0·18 |

computations, the normal method of dealing with $CO_2$ being obviously quite unsuitable for thin layers near the surface. Approximate treatment, on lines similar to the Kew chart for water vapour, suggests that the computed values of flux change should be increased by between 10 and 20 per cent.

## 7. The additional radiation

The present results confirm those of Part I in indicating the presence of a variable component of radiation of magnitude from 0 to about 10 per cent of the total atmospheric radiation measured at the surface. They give little further guidance as to the origin of this radiation, except to strengthen the opinion that it is almost certainly caused mainly by particulate matter in the atmosphere, *i.e.*, dust or tenuous cloud. It was suggested in Part I, section 13, that there might be some connection between the additional radiation and the origin of the air mass in the troposphere and lower stratosphere, but the present results do not support this. On the one hand cases with large additional radiation occurred in deep polar air, on the other the " minimum radiation " case of 28 May 1947 had very warm and stable air with a high tropopause.

As noted in Part I in many of the cases there was a tendency for excess emissivity to increase with increasing zenith angle, whilst in others the excess appeared to be independent of path length. Examples of both these types occurred in the new observations, and it now seems probable that the first type is caused by atmospheric pollution in the lowest (warmest) layers, whilst the second is due to very tenuous and uniform high cloud. This latter condition is extremely difficult for an observer at the surface to detect, particularly at night, and the experience of high-flying aircraft suggests that it is by no means a rare phenomenon.

## 8. Some anomalous observations

One series of observations, taken on 11 October 1946, is markedly different from all the others. This was mentioned in Part I, but the details were not given as it was hoped that subsequent experience might produce confirmation. No

similar case has occurred however, and the results will now be reported although no explanation can be suggested. The readings were obtained when only the Dines radiometer was in use, so that the possibility of an instrumental cause cannot be ruled out entirely. It is however thought most unlikely, as the Dines instrument appeared to be generally quite as reliable as the Linke-Fuessner, and was checked against the latter on 3 October 1946 and 19 October 1946.

The wind was easterly (*i.e.*, from the direction of London) throughout the period of observation, and conditions were notably hazy, though not more so than on several other days when normal results were obtained. There was no cloud until the formation of low stratus put an end to the measurements at about 2300 G.M.T. Fig. 8 shows how the total radiation varied during the period of measure-

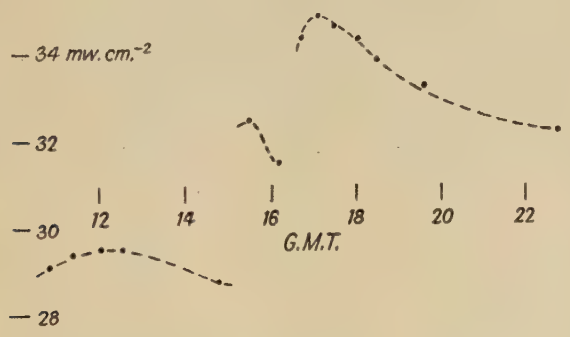


Figure 8. Downward flux of atmospheric radiation at the surface 11 October 1946.

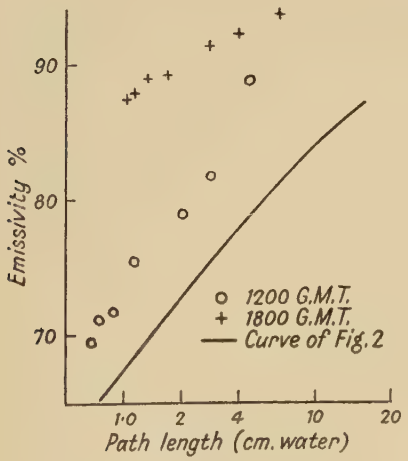


Figure 9. Variation of atmospheric emissivity with path length 11 October 1946.

ment. There was nothing unusual about the readings prior to 1500 G.M.T., computation by the Elsasser chart for 1200 G.M.T. giving a value of 29·5 mw $cm^{-2}$. Synoptic charts do not suggest any change of air mass during the afternoon, and computation of the 1800 value (Elsasser) yields 29·8 mw $cm^{-2}$. There is thus no obvious explanation of the great increase of radiation which occurred around 1600 G.M.T. Fig. 9 shows a plot of the emissivity against path length (before correction for $CO_2$, *i.e.*, analogous to Fig. 10, Part I) for the 1200 and 1800 G.M.T. observations. The former is seen to be quite a normal observation except for the presence of a large additional radiation of the type attributed to pollution in the lowest layers. The latter curve is of a type unique in the series, which it seems possible to explain only by the presence of considerable quantities of some polluting gas with strong absorption in the 7 to 15 $\mu$ region. The unobserved formation of a sheet of cloud could produce a similar effect, but this certainly did not occur, and the haziness both in the zenith and near the horizon decreased rather than increased during the experiment. A search was made of the Kew records to see if any other meteorological element showed a large change about 1600 G.M.T. The time was a little before sunset so that both solar radiation and temperature were falling rapidly and, because of the haze, rather irregularly. There was however no unambiguous indication of sudden change.

## 9. Surface temperature and the outward radiation

For many purposes the net flux of radiation is of more interest than the downward component, and the upward flux at the surface is of importance in studies of the transfer of heat and water vapour between ground and air. As we may assume the surface to be a close approach to a black body, the radiometer also provides a method of measurement of the mean value over a small area of the true surface temperature (" skin temperature "). A more simple means of measurement of this quantity would be advantageous, and in Part I it was reported that the temperature indicated by a grass minimum thermometer lying on the surface had been found to agree closely with the radiative temperature in darkness, diffuse daylight — *i.e.*, with cloudy skies, or on surfaces in shadow — and in sunlight with the sun's elevation less than 10°. Subsequent experience has confirmed this, and more than 150 readings are now available in these conditions. If $T_r$ is the radiative surface temperature and $T_s$ the reading of the surface thermometer it has been found that

$$T_r = T_s + 0{\cdot}25 \pm 0{\cdot}6 \ ^\circ\mathrm{C}$$

The readings include examples of all surface conditions — dry, wet, frozen, and snow covered.

A number of results have now been obtained in conditions of strong insolation. These readings proved rather more difficult to make than was anticipated, owing to fluctuations in short wave radiation, and probably also in surface temperature, of period long compared with the lag coefficient of the instrument (about two seconds) but short compared with the time required to make an observation in

daylight (about two minutes). Conditions were however reasonably steady on a number of days, and the results are thought sufficiently interesting to justify detailed report as in Table VI. These readings were all obtained on a short grass surface with a good cover of vegetation in both saturated (after rain) and dry conditions. With the previous notation the 30 readings gave

$$T_r = T_s - 0{\cdot}5 \pm 1{\cdot}2 \ ^\circ\text{C}$$

TABLE VI—RADIATIVE AND SURFACE THERMOMETER TEMPERATURES IN STRONG INSOLATION

| Thermometer readings °A | | Radiative temperature °A | $T_r - T$ °A | $T_r - T_s$ °A |
|---|---|---|---|---|
| Screen ($T$) | Surface ($T_s$) | ($T_r$) | | |
| 305 | 321 | 322·7 | 18 | +1·7 |
| 298 | 310 | 311 | 13 | +1·0 |
| 288·5 | 301·2 | 300·8 | 12 | —0·4 |
| 297·5 | 311 | 309 | 11 | —2·0 |
| 288 | 298·5 | 299·3 | 11 | +0·8 |
| 290 | 300 | 301·3 | 11 | +1·3 |
| 290 | 299·8 | 299·7 | 10 | —0·1 |
| 298 | 311 | 308 | 10 | —3·0 |
| 290 | 299·8 | 299·2 | 9 | —0·6 |
| 290 | 299·0 | 298·2 | 8 | —0·8 |
| 290 | 298·5 | 297·8 | 8 | —0·7 |
| 294 | 302 | 301·5 | 7 | —0·5 |
| 289 | 296 | 296 | 7 | 0 |
| 278·5 | 286 | 284·6 | 6 | —1·4 |
| 290 | 297·8 | 295·5 | 5 | —2·3 |
| 295 | 302·5 | 300 | 5 | —2·5 |
| 286 | 289 | 290 | 4 | +1 |
| 303 | 308 | 306·5 | 3 | —1·5 |
| 303 | 305 | 305·7 | 3 | +0·7 |
| 303 | 308 | 306 | 3 | —2 |
| 297 | 301 | 300 | 3 | —1 |
| 303·8 | 306·5 | 305·7 | 2 | +1·2 |
| 303 | 305 | 305 | 2 | 0 |
| 303 | 306 | 305 | 2 | —1 |
| 301 | 302·5 | 303 | 2 | +0·5 |
| 289 | 289 | 290 | 1 | +1·0 |
| 300 | 301 | 301 | 1 | 0 |
| 299 | 299·5 | 298·5 | 1 | —1 |
| 303 | 307 | 303 | 0 | —4·0 |

The mean values of $T_r - T_s$ in the above formulae can be attributed as to sign, and approximately as to magnitude, to the change in radiative flux between the surface and the level of the radiometer (normally about one metre), as explained in paragraph 6. It can be concluded that for a short grass surface the readings of a grass minimum type thermometer laid on the surface are usually a close approach to the true surface temperature, the standard deviations being about 1·5°C in strong sunlight and 0·75°C in other conditions. In four of the "sunlight" cases the individual deviations were much larger than the mean. In the most marked of these the temperatures were not checked as the readings were incidental to other work, and a 5° error in reading may have been made. The other cases are means of a number of readings in which the temperatures were checked, and we must conclude that the method will occasionally yield results in error by 2° — 3°C. The method might be of considerable use in studies of the "heat balance" type under field conditions when the measurement of long wave radiation is for any reason impracticable. The errors in radiative flux corresponding to the standard deviations are about 5 per cent of the total flux of both solar and atmospheric radiation. It is also most desirable that surface temperature should be observed

in any micro-meteorological study, preferably measured radiometrically, but if this is not possible, by the present method. It must be emphasised that the tests have been made exclusively on a short grass surface. It may also be advisable to remark that the differences between screen and surface temperature shown in Table VI are not necessarily typical of the differences existing on a sunny day in summer. The higher values of this difference were usually associated with large fluctuations and often could not be measured satisfactorily. Table VI is therefore believed to contain less than the normal proportion of large differences.

## 10. Some comments on the value and validity of radiation chart computations

The main purpose of the experiments here described has been to check the validity of the radiation chart. It is important to realise the limited nature of the tests which have been made, and for this purpose it is proposed to conclude with a discussion of two possible applications of the chart to meteorological problems, and to show how far the present observations support its use in these particular cases.

Synoptic studies have shown that temperature changes due to the radiative and convective transfer of heat may be of considerable importance in determining the movement and development of pressure systems, and the radiation chart appears to be a powerful and convenient method of investigating such changes. When the chart is used in this way particular importance is attached to the results for the middle and upper troposphere, especially if there is cloud at these levels. The flux of radiation at the tropopause also becomes a matter of interest. Now the present experiments afford very little evidence to support the use of the chart in this way, since more than 90 per cent of the radiation which has been studied originated in the surface layers, at pressures greater than say 950 mb and temperatures between —5 and +35°C. The pressure variation of emissivity is therefore without influence on the results ; and the conclusions of section 5 concerning the temperature variation hardly induce great confidence in the results of either Elsasser's or the Kew chart at low temperatures. On the other hand 5 to 10 per cent of radiation from the upper levels is included in the results, and the general consistency, particularly of the ' minimum radiation ' cases, suggests (but no more than suggests) that in the upper levels the charts are unlikely to be in error by a factor of more than $\times$ 2. Further confirmation is needed, and seems to be possible in two ways. One is by measuring from aircraft the total flux of radiation — quite feasible using modern techniques — the other is by laboratory investigation of the emissivity of air and water vapour at low temperatures and pressures. It is believed that detailed investigation of the infra-red spectra of atmospheric gases is in progress in more than one laboratory and it would be gratifying if the meteorologically important total emissivity could be included in these studies. In the meantime considerable caution must be exercised in applying the radiation charts to upper atmospheric data.

A second possible application of the charts is to the problem of heat exchange between the surface and the lowest layers of air, with its bearing on diurnal variations of temperature and the incidence of frost and fog. Here there is the practical difficulty of setting a value to the effective radiative temperature of the surface, but where this does not exist, *e.g.*, over the uniform short grass surface of the present experiments, the results are considered to justify the use of the charts, particularly the Kew chart. The method is however not very convenient, and the need to determine long and very narrow areas limits its precision. Experience suggests that the results should be trustworthy to within at worst $\pm$ 25 per cent after making approximate allowance for the effect of $CO_2$. The experiments of section 6 are a direct test of this application, and the precision of the chart can be determined only by the extension and refinement of such measurements. A word of warning must be added. If computations of this type are made using high surface temperatures (*e.g.*, 40 °C — a not unusual summer value for the lawns at Kew) it is found that the Elsasser chart yields values 30 to 40 per cent below those of the Kew chart. No explanation can be suggested, for although at these temperatures the Elsasser emissivities are less than those used in the Kew chart the difference would not be expected to have such a large effect on the computations.

## Acknowledgments

I wish to thank Dr. J. M. Stagg, formerly Superintendent of Kew Observatory, for his interest and advice, and the Director of the Meteorological Office for permission to publish the results.

## References

Cowling, T. G. 1943 *Reports on Progress in Physics*, London, **9**, p. 29.
Elsasser, W. M. 1942 *Harvard Met. Studies*, Milton, Mass., No. 6.
Robinson, G. D. 1947 *Quart. J.R. Met. Soc.*, London, **73**, p. 127.
1948 Unpublished communication to the Meteorological Research Committee (*M.R.P.* 377).

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# The composition of coagulation-elements in cumulonimbus

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SUMMARY

Calculations are made from which it appears that the rate of loss of heat from ice particles growing by coagulation in dense supercooled clouds may be insufficient to allow all the acquired water to freeze. The larger coagulation-elements within cumulonimbus clouds may therefore have liquid skins at levels where the temperature has fallen to —20° or —30°C. The rain-drop multiplication process envisaged by Langmuir may thus proceed also at temperatures well below 0°C, and the effect may also be important in the production of ice nuclei by ' splinter ' formation and in the generation of thunderstorm electricity.

The researches of Findeisen and Schulz (1944) have shown that in the rapidly rising air within large cumulus clouds the number of ice particles formed upon ice nuclei is very small compared with the number of droplets at least until the temperature has fallen into the neighbourhood of —32°C. Before this temperature is reached the number of active ice nuclei is less than 1/litre, whereas the number of droplets is of the order $10^5$/litre. In these circumstances the growth of each of the ice particles is unrestricted and may be estimated by making suitable assumptions concerning their density and falling-speed, and the liquid water content of the parts of the cloud through which they fall. The purpose of this note is to suggest that the larger ice particles may have wet surfaces with outside air temperatures as low as —20° or —30°C.

Rough calculations show that in a supercooled water cloud of liquid water content 1 g/m$^3$ once an ice particle has attained a diameter of about one millimetre its rate of growth by coagulation with droplets in its path already exceeds that due to the direct condensation of vapour, and beyond this stage the growth due to coagulation accelerates to become of overwhelming importance, both the volume swept by the particle and its rate of fall rapidly increasing. In stratiform clouds of small liquid content water (considerably less than 1 g/m$^3$) the development of large snowflakes shows that the ice particle growth proceeds largely by direct condensation, but within large cumuliform clouds in the early stages of the development of precipitation the liquid water content may be expected commonly to exceed 1 g/m$^3$, and therefore in these clouds ice particles of diameter greater than one millimetre may be regarded as coagulation-elements whose further growth is due solely to coalesence with droplets in their falling-path.

Investigations by the writer, not yet published, indicate that within large cumulus clouds (of depth exceeding some 2000 m) the rising air masses ascend at a high speed (of the order of 10 m/sec) and cool at about the saturated adiabatic rate with respect to liquid water. Inside such clouds, therefore, the liquid water content at any level may easily be read from an aerological diagram ; for clouds having bases at the same level this content increases with the temperature at the cloud base, and Fig. 1 shows the values obtained for clouds with bases at the 900 mb level. It will be seen that the liquid water content above the so-called freezing level varies from about 1 g/m$^3$ (in cold weather) to about 7 g/m$^3$ (in warm weather).

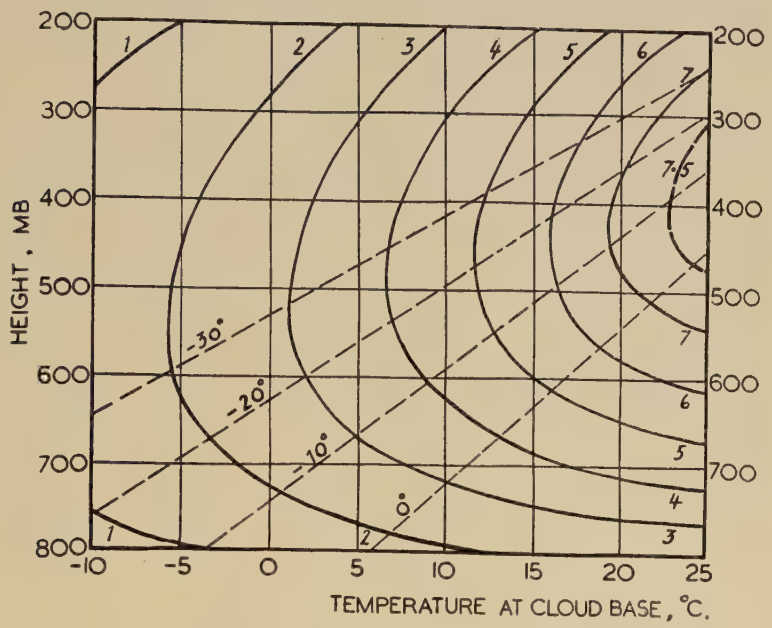



Figure 1. Liquid water content of cumuliform clouds. The values are calculated for clouds having bases at 900 mb ; saturation with respect to liquid water is assumed throughout and the isopleths show the concentration of liquid water in g/m³. The pecked lines indicate the temperature within the clouds.

The coagulation-process gives the growing ice particles a coat of supercooled water, which commences to freeze at the ice surface. Here a rise of temperature occurs as the latent heat of fusion is liberated, and the temperature at the ice surface and within the liquid film tends to rise to 0°C. If the coagulation-element could be thermally insulated its temperature would quickly reach 0°C, and only a small fraction of the supercooled water added during collisions could become ice. However, the coagulation-element loses heat to the environment, partly by conduction and partly by the evaporation of some of the water from its surface. The rate at which the mass of the ice particle can increase depends upon the rate at which this loss of heat can proceed, and this is governed by many factors, including the size and shape of the falling particle, and its speed and temperature relative to the surrounding air. At low temperatures and when the rate of coagulation is small, the supercooled droplets striking the ice particles may be frozen almost at once, and the coagulation-product then grows as a roughly spherical pellet of granular ice, containing air enclosures and appearing opaque, which is the typical form of precipitation from small cumulonimbus in cold weather. When the rate of coagulation is high, however, it may happen that the supercooled water added to the ice particle accumulates faster than it can be frozen, and the coagulation-product may eventually consist largely of water, while the embedded ice particle acquires a sheath of hard and transparent ice.

The rate at which a spherical particle of radius $r$ loses heat while falling through air at a lower temperature is

$$H_1 = a \cdot 4\pi r^2 \cdot \Delta T,$$

where $\Delta T$ is the difference in temperature between the surface of the sphere and the undisturbed air, and $a$ is a coefficient whose value varies with the Reynolds's Number and is given approximately (Kramers 1946) by the following relation :

$$2r \cdot \frac{a}{K} = 3{\cdot}2 + 0{\cdot}59\ (Re)^{\frac{1}{2}},$$

where $K$ is the coefficient of conductivity of heat of the air.

To simplify calculations we may put $\theta = \frac{a}{K}$, so that

$$H_1 = 4\pi\ (\theta r^2)\ K\ \Delta T \quad . \quad . \quad . \quad (1)$$

The rate of evaporation from a wet sphere in moving air has been measured by several experimenters, and the most reliable measurements for our purpose appear to be those of Frössling (1938), who gives the following relation :

$$dw/dt = 4\pi k r \cdot \Delta p.\ C,$$

where $dw/dt$ is the mass evaporated in g/sec,
$k$ is the coefficient of diffusion of water vapour,
$\Delta p$ is the difference in vapour density (g/cm$^3$) between the drop surface and the undisturbed flow, and
$C$ is a coefficient determined by the relative air speed according to the relation :

$$C = 1 + 0{\cdot}229\ (Re)^{\frac{1}{2}}$$

The rate of heat loss due to evaporation is then

$$H_2 = 600\ dw/dt,$$
$$= 600 \cdot 4\pi k r \cdot \Delta p.\ C,$$

taking the latent heat of evaporation to be 600 cal/g, or

$$H_2 = 2400\pi k\ (Cr)\ \Delta p \quad . \quad . \quad . \quad (2)$$

Adding (1) to (2) we find for the total rate of heat loss*

$$H_1 + H_2 = 4\pi\ (\theta r^2)\ K\Delta T + 2400\pi k\ (Cr)\ \Delta p \quad . \quad . \quad (3)$$

Now in unit time the coagulation product falling at $v$ cm/sec sweeps a volume $\pi r^2 v$, so that assuming that complete coalescence occurs,† the rate of coagulation is $\pi r^2 v m$ where $m$ is the concentration of liquid water in the cloud. If the added

* Some heat is conducted from the surface layer into the interior of the coagulation-element ; a simple calculation, however, shows that the heat required to raise the temperature of the entire element by 30°C is only about equal to that disposed of to the environment in a few seconds by the wet surface, and it has therefore been neglected in the present estimations.

† The computations of Langmuir (1948) indicate that this is a reasonable assumption for coagulation-elements having radii greater than $10^{-1}$ cm : in dense clouds assumed to contain droplets of radii predominantly greater than $10\mu$, these assimilate more than 90 per cent of the water content of the volume swept. Smaller elements assimilate a lesser proportion, a factor helping to keep their surfaces dry.

water is to be frozen completely, then a quantity of heat

$$H_3 = \pi r^2 v m \ (80 + T). \qquad . \qquad . \qquad . \qquad (4)$$

has to be removed, where $T$ is the temperature of the added water in °C, *i.e.*, the temperature of the outside air.

Using the above relations we may determine whether the maximum rate of loss of heat from the coagulation element is sufficient to allow the complete freezing of the water gained by coagulation. Clearly the maximum rate of heat loss occurs when the surface of the coagulation product is a film of liquid water at 0°C and both $\Delta T$ ($=T$) and $\Delta p$ (= the difference between the vapour concentrations corresponding to saturation at 0°C and at the outside temperature) in the above expression for $(H_1 + H_2)$ have their maximum values. Evidently under natural conditions a surface film of liquid water will begin to accumulate before the attainment of the critical conditions which we shall calculate on the above assumption, and reliable information on the composition of the coagulation product can only be obtained experimentally ; but the purpose of our calculations is to show that the critical conditions may often occur within cumulonimbus clouds, and that within a certain range of temperature the larger coagulation products are likely to contain a high proportion of liquid water.

Table I shows the values of the falling-speed,* $(Re)^{\frac{1}{2}}$, $(\theta r^2)$ and $(Cr)$ which we have used in the following calculations :

TABLE I

| Droplet radius cm | Falling-speed cm/sec | $(Re)^{\frac{1}{2}}$ | $(\theta r^2)$ | $(Cr)$ |
|---|---|---|---|---|
| $10^{-2}$ | 73 | 3 | $2{\cdot}5 \times 10^{-2}$ | $1{\cdot}7 \times 10^{-2}$ |
| $5{\cdot}10^{-2}$ | 400 | 14 | $2{\cdot}9 \times 10^{-1}$ | $2{\cdot}1 \times 10^{-2}$ |
| $10^{-1}$ | 650 | 26 | $9{\cdot}2 \times 10^{-1}$ | $7{\cdot}0 \times 10^{-1}$ |
| $2{\cdot}10^{-1}$ | 850 | 42 | 2·8 | 2·1 |
| $3{\cdot}10^{-1}$ | 1250 | 54 | 5·1 | 4·0 |
| $5{\cdot}10^{-1}$ | 1600 | 79 | 12·5 | 9·6 |

The coefficients $K$ and $k$ have been calculated from the following relations ;

$$K = (573 + 1{\cdot}8T) \times 10^{-7} \qquad k = {\cdot}239 \left(\frac{273 + T}{281}\right)^2 \times \frac{1013}{p},$$

in which $T$ is temperature in °C and $p$ is pressure in mb.

We now consider the conditions in two clouds having bases at the 900 mb level and at temperatures of 10°C and 20°C. In each cloud the values of temperature and liquid water content at higher levels provide a set of conditions under which the maximum value of the expression for $(H_1 + H_2)$ in Eq. (3) and the value of $H_3$ in Eq. (4) may be calculated for coagulation elements of different sizes. By equating these values of $(H_1 + H_2)$ and $H_3$ for each element and solving for $m$, we obtain a critical value for the liquid water content of the cloud, at which the

* The falling-speeds for radii less than $10^{-1}$ cm are approximately those of droplets at ordinary temperatures and pressures, which are sufficiently accurate for our purpose in view of the opposing effects of the smaller density of the ice particles and the smaller air densities at the high levels under consideration. For the other radii falling-speeds have been estimated for the 500 mb level, assuming the particles to be spheres of density 0·8 and with a drag-coefficient of 0·5.

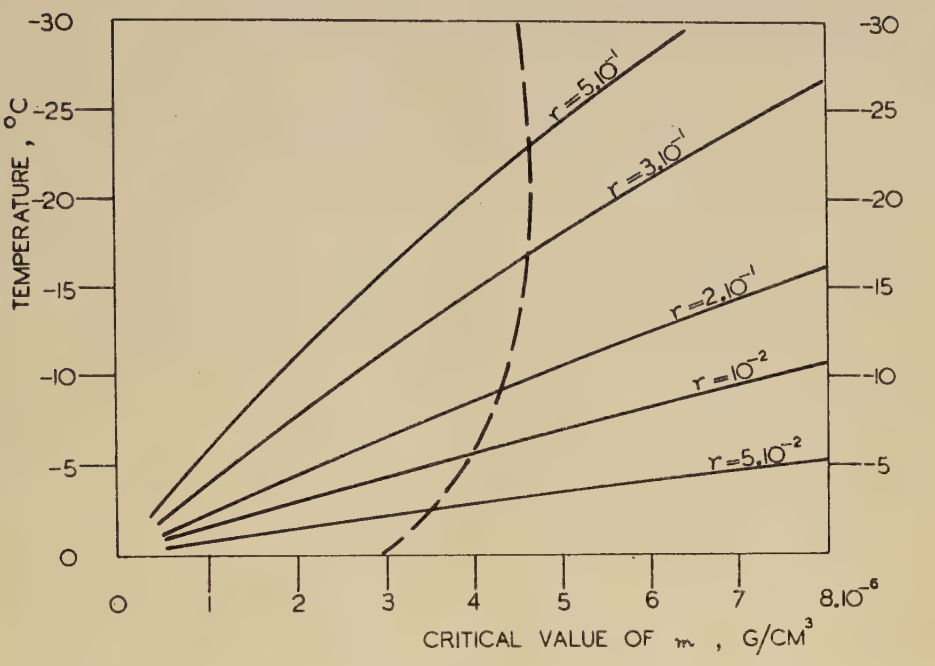
maximum rate of heat loss is just that required to allow all the coagulated water to freeze. Figs. 2, 3 show the critical values of $m$ for coagulation elements of different sizes at air temperatures between 0°C and —30°C in the two clouds. The superimposed pecked lines show the actual values of $m$, transferred from Fig. 1. If for a particular coagulation element the actual value of $m$ exceeds the critical value corresponding to its radius, then liquid water must accumulate on the element. The pecked lines therefore divide the other lines on the figures into two parts : to the left the elements acquire liquid skins, while to the right they remain wholly solid.

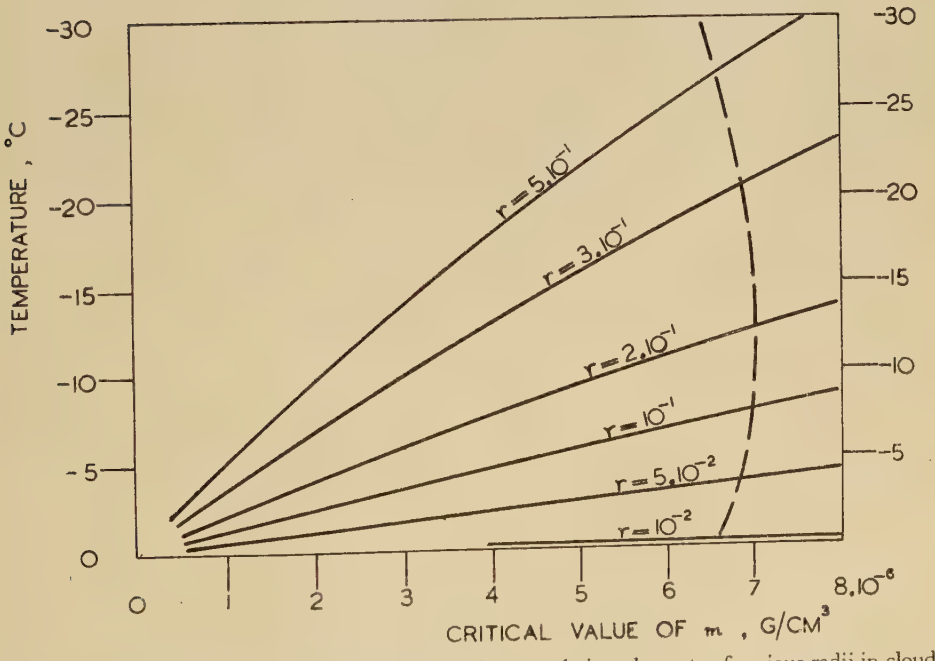
An inspection of these figures shows that except in a very narrow zone where the temperature is close to 0°C small coagulation elements (of diameter about one millimetre) are always able to assimilate the acquired water and grow as completely frozen opaque structures. If the liquid water content of the cloud is low the same is true of the larger elements ; this is in accord with the experience that soft hail falls from cumulonimbus in cold weather and is also found beneath small cumulonimbus having bases in the medium levels.

More striking is the conclusion that if the liquid water content of the cloud is high the larger coagulation-elements accumulate liquid water at temperatures as low as —20° or —30°C. In these circumstances it is doubtful if the coagulation-products can exceed a diameter of about 0·6 cm without the surface liquid becoming disrupted and shedding small water drops. The effect of this process would be to reduce considerably the rate of growth of the coagulation-elements when they have attained diameters of about 0·6 cm and falling-speeds of about 12 m/sec. The droplets which are flung off will themselves grow by coagulation, and of course wholly as liquid water unless they happen to contain freezing nuclei or become infected with growing ice particles by coagulation. Thus numbers of droplets may be carried up to levels where the temperature is —30°C or even less, and when the up-draughts exceed about 10 m/sec large coagulation-elements with wet surfaces may also reach these levels ; the chain reactions of drop multiplication described by Langmuir (1948) may therefore also occur in these upper parts of the cloud.

The presence of alternate layers of clear and opaque ice which is often noticed in hailstones must be regarded as due to successive passages through the critical conditions caused by changes in radius, falling-speed, temperature, pressure and liquid water content in the cloud. The core of a hailstone will always consist of opaque ice ; subsequently a layer of clear ice will be added before a further opaque growth occurs higher in the cloud at very low temperatures, and a thin clear layer may be formed during fall through the levels where the temperature is close to 0°C. Four distinct layers should therefore be a common occurrence. It is not necessary, as formerly supposed, to explain these layers by fluctuating up-currents which cause the hailstones to make several return journeys between the upper and lower parts of the cloud.

When the temperature within the ascending cloud-air falls into the neighbourhood of —32°C the number of ice particles formed (both by the action of freezing




Figures 2, 3. Critical liquid-water concentrations for coagulation-elements of various radii in clouds with bases at 900 mb and base temperatures of 10° and 20°C respectively. If at a particular temperature the actual concentration (shown by the pecked lines) exceeds the critical value, then liquid water accumulates on the coagulation-element.

nuclei and by the formation of the ice splinters, described below, during the previous few minutes) becomes comparable with the number of droplets present, and the calculations which we have made above are no longer valid, since in these it was assumed that there was no limitation to the amount of liquid water available. Now, however, growth by coagulation in this uppermost zone must soon dwindle greatly, and vast numbers of small ice particles are grown ; these will be wholly solid or crystalline structures, but while a powerful up-draught persists wet hail and both large and small water drops may arrive from beneath, so even this part of the cloud may at first contain a considerable amount of liquid water.

The predominant presence of wet coagulation-elements at temperatures down to about —30°C may be of interest in the study of radar echoes from cumulonimbus, and may have an important bearing on the production of thunderstorms. According to Findeisen (1940, 1943) the development of thunderstorm electricity is due largely to processes attending the growth or evaporation of ice particles during their fall. He has found that air flowing past the ice particles carries away minute " splinters " of ice (radius about 20 $\mu$) which are electrically charged, negative charges predominating when the ice particles are growing. As a result the larger ice particles gain a positive charge during their growth, which may be increased during their melting by the liberation of air bubbles (Dinger and Gunn 1946), and also by the breaking away of any droplets during disruptions. This " splinter-formation " is of great importance in the ice nucleus economy of clouds, particularly of mixed clouds, since it is most pronounced during growth by coagulation. However, it occurs only when the ice particles consist of aggregates of smaller crystals : it does not occur with simple crystals and ceases if the ice particles acquire a glassy or wet surface. According to Findeisen the rate of splinter formation is proportional to the surface area of the ice particles, and the splinters therefore arise predominantly from the ice particles of larger radius, which, however, are those likely to acquire wet skins at temperatures above about —30°C. Since at temperatures below about —32°C the number of ice particles arising is for the first time such that they must grow more slowly as wholly solid and dry structures, this temperature may be a critical summit temperature in the production of thunderstorms by cumulonimbus clouds.

## Acknowledgment

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## References


| | | |
|---|---|---|
| Dinger, J. E. and Gunn, R. | 1946 | *Terr. Mag.*, Baltimore, Md., **51**, p. 477. |
| Findeisen, W. | 1940 | *Met. Z.*, Braunschweig, **57**, p. 201. |
| | 1943 | *Ibid.*, **60**, p. 145. |
| and Schulz, G. | 1944 | *Forsch. u. Erfahrb. des Reichs. Wetterd.*, Berlin, Ser. A, Nr. 27. |
| Frössling, N. | 1938 | *Beitr. Geophys.*, Leipzig, **52**, p. 170. |
| Kramers, H. | 1946 | *Physica*, Hague, **12**, p. 61. |
| Langmuir, I. | 1948 | *J. Met. Amer. Met. Soc.*, Boston, Mass., **5**, p. 182. |

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# The nature of ice-forming nuclei in the atmosphere

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SUMMARY

During the last decade many experiments have been designed to investigate the initial processes of ice formation in the atmosphere. The results of these experiments are examined for clues as to the identity of the responsible nuclei and the following tentative conclusions are reached :

(*a*) The nuclei which cause freezing between 0°C and —32°C are mainly solid, insoluble particles which are wetted by water and produce ice particles through the freezing of water drops.

(*b*) The nuclei which become operative in the range —32° to —41°C consist of droplets of sea salt solution, ice and salt crystallising out on contaminating foreign particles at a temperature depending on the efficiency of these particles and the rate of expansion of the ambient air.

(*c*) The nuclei effective at just below —41°C consist of droplets of *pure* salt solution and possibly of " gaseous " nuclei formed industrially or by the action of ultra-violet light on the gases of the upper atmosphere.

## 1. EARLIER THEORIES

### (a) *Sublimation-Nucleus Theory*

Wegener (1911) first suggested that ice crystals were formed by the direct sublimation of water vapour on to sub-microscopic solid particles. This view was supported by Findeisen (1938) and Simpson (1941), who suggested that the sublimation nuclei might be particles of quartz, which because of its hexagonal crystal structure, was assumed to be nearly isomorphous with ice. In a later paper Findeisen (1944) claimed evidence for the existence of two distinct kinds of sublimation nuclei. The conditions for isomorphism, however, are quite strict and X-ray analysis has revealed not only considerable dissimilarity in the crystal structures of quartz and ice, but also that the bonding of the crystal elements in the lattice is of a different nature in the two substances. Further, the introduction of quartz dust into an expansion chamber (Regener 1941) and on to the surface of a condensation plate (Weickmann 1942), does not cause a significant increase in the number of ice particles. Recent experiments, as we shall see, indicate that true sublimation nuclei are very rare in the atmosphere.

### (b) *Krastanow's Classification of Nuclei — Soluble and Insoluble Nuclei*

Krastanow (1940, 1941), has classified atmospheric nuclei into two main types according to whether or not they are wetted by water. He assumes the wettable nuclei to be enveloped by a water skin at high relative humidities and proceeds to treat them as liquid droplets of equivalent size. These nuclei are subdivided into two groups, soluble and insoluble.

The majority of the soluble nuclei are considered to consist of small droplets of salt solution and to persist as liquid down to temperatures well below 0°C.

A solid insoluble nucleus wetted by water will preserve its identity inside the liquid droplet, and Krastanow suggests that, if the temperature is lowered sufficiently, an ice-bud may form on the surface of the particle from the surrounding super-cooled water. The criterion will be the work required for ice-bud formation which will depend largely upon the size of the particle, its crystal and surface structures, and the surface energy of the liquid/solid interface. If the particle is bounded by surfaces of different size and structure, ice-buds will form first on the surface for which the energy required is least.

In general, then, all nuclei capable of being moistened by water form droplets initially, but under suitable conditions of temperature and humidity, depending on their size, shape and surface structure, can act as centres of ice formation and cause super-cooled drops to freeze.

Of the nuclei not wetted by water, those which are isomorphous with ice are of particular interest as, at a temperature and vapour pressure determined by the nuclear size, ice crystals will grow upon them by direct sublimation of the vapour. Such nuclei, however, are considered by Krastanow to be extremely rare.

Other non-wettable nuclei, not isomorphous with ice, are unlikely to act as sublimation nuclei unless they possess steps or cracks on their surfaces, but as most substances are wetted by water, such nuclei will be comparatively rare.

Krastanow concludes that most atmospheric nuclei taking part in condensation processes are wetted by water, so that droplets are generally formed as a first product, ice crystals being produced at lower temperatures when the drops freeze. He proceeds on the basis of earlier work by Becker and Döring (1935), and Volmer (1939), to determine theoretically the sequence of droplet and ice crystal formation in relation to the temperature, water vapour pressure, and the size of the nuclei. The theory employs the methods of statistical physics and makes the fundamental assumption that the type of bud requiring the smaller vapour pressure for its formation will be favoured.

He first deals with the spontaneous formation of ice crystals from homogeneous water vapour (no nuclei present) and calculates that they can appear as primary products below —65°C, but at higher temperatures liquid drops are always formed. This calculated transition temperature depends, however, on the values assumed for the surface tensions of super-cooled water and ice which so far have not been determined. In any case the physical meaning of these concepts when applied to small aggregates of molecules is by no means clear.

Krastanow next considers the highly artificial case of an atmosphere containing wettable nuclei treated as droplets of equivalent size and insoluble, non-wettable, nuclei of the same size but bounded by regular faces isomorphous with ice. He calculates the temperature and vapour pressure which fix the limits between the formation of droplets on the first type of nuclei and the formation of ice crystals by sublimation on the second type.

His results indicate that, even in the presence of ideal sublimation nuclei, super-cooled water drops are formed as first products more easily than are ice crystals by sublimation, as it requires an additional amount of work to form a crystal

bud on the surface of the solid nucleus from the vapour phase. For larger nuclei this work is reduced, so that ice can form at a higher temperature. It also follows that, as the interfacial surface energy between the solid particle and the super-cooled water is less than that between the solid and vapour, ice crystals will arise more easily from super-cooled drops containing solid particles than from the vapour phase by direct sublimation.

In order to carry out such a calculation, Krastanow has to make many simplifying assumptions. It is assumed that the nuclei involved have a simple, regular shape and are isomorphous with ice. Only guesses can be made as to the magnitude of several physical parameters, *e.g.*, the concentration of effective nuclei, the rate of bud formation, and the surface energies of the various interfaces involved. For these reasons the actual quantitative results are of doubtful value. Also, since the physical properties of the nuclei actually occurring in the atmosphere are in general unknown, Krastanow's theory cannot be applied immediately to determine the sequence of droplet and ice crystal formation under conditions which are meteorologically important. It should be regarded as expressing the broad physical principles of the problem and as indicating the general method of attack, rather than as being capable of yielding reliable quantitive results which are relevant to the atmosphere.

## 2. Summary of recent experimental work

Excellent reviews of some of the modern researches have been presented in recent numbers of the *Quart. J. R. Met. Soc.* by Ludlam (1948) and Dobson (1949). The results of experiments described in their papers will therefore be summarised in tabular form only, more attention being given to experiments which have not been discussed in the current literature.

The results which emerge from the many experiments show many common features and point to the existence of three main types of ice-forming nuclei in the atmosphere. They are characterised by the temperature ranges over which they are effective, *viz.*: 0°C to —32°C, —32°C to —41°C, and just below —41°C. In addition there is a spontaneous formation of ice from homogeneous water vapour which appears to occur at about —70°C. The experimental results are therefore summarised under these four headings.

### *The Experiments of Hollstein*

The work of Hollstein (1947) seems to have attracted little attention among meteorologists but is very relevant to the problem under discussion.

She has investigated the super-cooling of common salt solutions of various concentrations contained in small ampoules of about 1 $cm^3$ capacity. The freezing points of these solutions were always lower than the corresponding values for sodium chloride quoted in the *International Critical Tables*, by a constant amount of about 14°C — the same temperature interval through which it was possible to super-cool the distilled water used in making up the solutions. These results are exhibited in Fig. 1, curve (a) being plotted from the tables, and curve (b) from Hollstein's results.

EXPERIMENTAL RESULTS—PROPERTIES EXHIBITED BY NUCLEI OPERATIVE OVER TEMPERATURE RANGES

| Author | Experimental Method | 0°C to —32°C | —32°C to —41°C | Below —41°C |
|---|---|---|---|---|
| Meyer and Pfaff (1935) | Filtered out nuclei with collodion filters (pore diam. $10^{-6}$ cm). | Water so treated was supercooled to —24°C. | | |
| | Repeated distillation and filtration of water. | Supercooling prolonged to —33°C. | | |
| Weickmann (1942, 1947) | Microscopic examination of condensation products formed on metal plate in cooling chamber. | Residue of evaporated drops showed spots on which condensation occurred at 85 per cent R.H. Ice did not form on these even at —37°C.<br><br>Ice crystals formed only at water saturation and mainly at edge of evaporation ring. | In ordinary room air, only extremely few ice crystals formed on metal plate above —38°C at high ice-supersaturations. At water-saturation numbers considerably increased. | Number of drops creased when air heat by stove supplyi nuclei (probably $H_2SO_4$), but few crystals appear among them at te peratures down —39°C. |
| | Microscopic examination of melted cirrus particles. | Drops contained small solid insoluble particles about 1$\mu$ diam., deposited mainly at edge during evaporation of drop.<br><br>Residue showed hygroscopic spots. | | |
| Bigg (1944) | Microscopic examination of droplets formed on metal surface in cooling chamber. | Ice formation started at definite spots in drop. On alternate freezing and thawing of droplet, crystallisation occurred repeatedly at reproducible temperature from single nucleus, but after several repetitions, nucleus became sterile and was succeeded by new nucleus at lower temperature. Majority of nuclei caused freezing between —10°C and —12°C although a few acted at higher temperatures.<br><br>Maximum supercooling obtained in presence of nuclei was to —33°C.<br><br>Freezing nuclei, previously ineffective, became reactivated on drying off at edge of drop. Became sterile again when submerged in bulk liquid. | | |
| Findeisen and Schulz (1944) | Controlled expansions in large chamber — rate of expansion varied.<br><br>Dependence of temperature at which ice crystals form on rate of expansion investigated. | In range 0° to —10°C only occasional ice particle — number increased as temperature lowered — concentration reaching 1 per litre at —32°C. These nuclei produced ice crystals at *lower* temperature when rate of expansion was increased (Findeisen "Type 1").<br><br>A few nuclei formed ice crystals at high ice supersaturations but majority were ineffective until water saturation was reached. | Below —32°C sharp increase in number of ice particles occurred, concentration rising to about 1 per $cm^3$.<br><br>These nuclei promoted ice formation at *higher* temperatures when rate of expansion increased (Findeisen "Type 2").<br><br>Ice crystals appeared only with water drops (*i.e.*, at water saturation). | |

EXPERIMENTAL RESULTS—PROPERTIES EXHIBITED BY NUCLEI OPERATIVE OVER TEMPERATURE RANGES—*continued*

| Author | Experimental Method | 0°C to —32°C | —32°C to —41°C | Below —41°C |
|---|---|---|---|---|
| long 947, a, b) | Controlled expansions in small chamber. | No ice crystals observed above —32 °C ; apparent absence due to small volume of chamber. | Below —33 °C in outdoor air a few ice particles appeared among many water drops. Number increased gradually as temperature fell to —41 °C.<br>Experiments on top of Jungfraujoch and over mid-ocean showed nuclei in this range to be practically absent. | In outdoor air, large increase in ice particles below —41 °C. In air cleaned by repeated expansions and supposedly free of nuclei, ice particles still appeared below —41 °C, assumed to be formed by sublimation of vapour on to ions. (But see below)<br>Nuclei operative below —41 °C present on Jungfraujoch and over mid-ocean. |
| ilong 1949) | | | Observed concentration of these nuclei less at Wellington (N.Z.) when sea breeze blowing. | |
| rnier d'Albe 1949) | Controlled expansions in small chamber.<br>Microscopic observation of condensation products in very small volume. | | Below about —32 °C in outdoor air a few ice particles appeared with many drops. Critical temperature not definite — varied between —32 °C and —36 °C.<br>Nuclei absent in cleaned air | Below —41 °C, in outdoor air, fog composed almost entirely of ice particles but only when dew point reached.<br>Critical temperature reproducible to within 0.2 °C.<br>In clean air, no ice crystals formed below —41 °C. Is suggested that Cwilong's results were due to a few nuclei which escaped cleaning process. |
| | Introduction of chemically pure salts as artificial nuclei into chamber containing clean air. | AgI nuclei produced ice crystals below —7 °C but only after *dew-point* was reached. Thus behaved as "freezing" nuclei.<br>$CdI_2$ nuclei, in crystalline state produced ice crystals below —9 °C at the *frost-point*, so apparently behaving as true sublimation nuclei. | | Nuclei of pure NaCl, $NaNO_3$ and other salts produced ice crystals only below —41 °C and at water saturation. At higher temperatures only water drops were formed. |
| mer 1949) | Large chamber, method much same as for Findeisen and Schulz. | Nuclei found in range —18 °C to —32 °C, numbers increasing at lower end of range. Efficiency of nuclei showed same general dependence on rate of expansion as found by Findeisen and Schulz.<br>Occasional ice particles observed at high ice-supersaturations. | | |
| | As for Cwilong, but used in aircraft. | | Nuclei in this range found in surface air but absent above inversions, stratus cloud and haze top. | Nuclei operative below —41 °C found at all heights up to 6 km (ceiling of observations). |
| wer and almer (see obson, 1949) | As for Cwilong. | | | Confirmed Fournier d'Albe that no ice crystals appear even at —41 °C if air completely free of nuclei.<br>Nuclei operative at —41 °C produced by action of ultra-violet radiation. |

Now the left-hand branch of (a) is generally regarded as representing the temperatures at which salt solutions of various concentrations up to the eutectic concentration E, are in equilibrium with ice. If a solution of concentration less than eutectic is cooled ice will commence to separate as soon as its equilibrium temperature is passed. As ice separates, the solution becomes richer in salt and the freezing point eventually falls to E, the eutectic point, where the solution completely solidifies, the solid phases being ice and a salt hydrate $NaCl.2H_2O$. This eutectic temperature (—21·2°C) is commonly regarded as being the lowest temperature at which sodium chloride solutions can exist. If a solution of concentration greater than eutectic but less than that indicated by M, is cooled, the hydrate separates out causing dilution of the solution. This process will continue with cooling until the eutectic temperature is reached, when again only the solid phases will exist. Only when the initial concentration of the solution is greater than that represented by M and the initial temperature above + 0·15°C, is anhydrous sodium chloride deposited on cooling. For aqueous solutions of common salt,

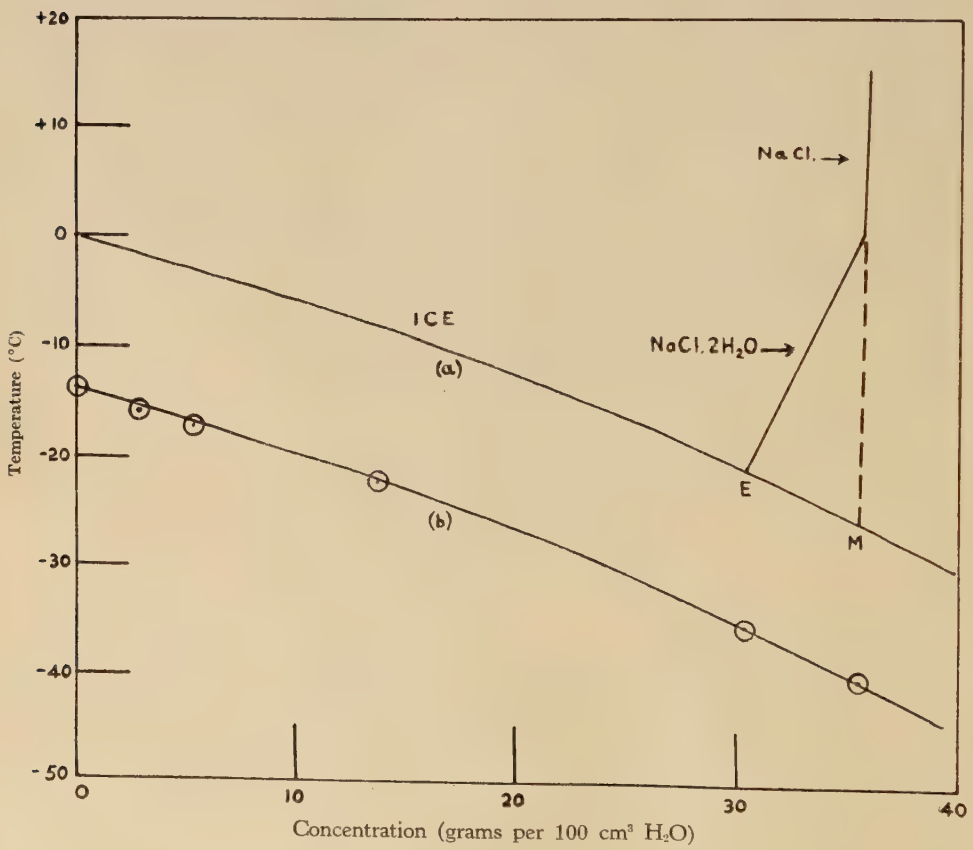


Figure 1. Phase diagram for sodium chloride solutions.

however, the eutectic point is generally the metastable one, M, for the anhydrous salt and ice.

Hollstein found however, that an eutectic solution of common salt could be super-cooled to —35·5°C and a solution saturated at 0°C to about —40°C.

A possible explanation of these results is as follows. If ice or salt is to separate out from a cooling solution, nuclei must be present on which the crystal-buds can form. If the solution consists of a pure salt and distilled water free from such nuclei, it would appear possible to super-cool it to very low temperatures, presumably until ice and salt crystallise out spontaneously (but see Section 3c). Freezing of the solutions at the temperatures shown in curve (a) would require the presence of freezing-nuclei operative at 0°C. Such nuclei must have been absent in Hollstein's samples, where the most efficient apparently became active at —14°C. These suggestions are supported by Rau's observations (Rau 1944) that very few freezing nuclei are effective at 0°C, the majority being operative between —10°C and —12°C.

Hollstein extended this work to observations on droplets of concentrated common salt solutions, sprayed on to the metal condensation surface of a Weickmann cooling-chamber. Most of the droplets crystallised during their fall, and crystals of sodium chloride were identified on the plate. When the temperature fell below —30°C however, a sudden change occurred, the water films on the majority of the crystals becoming dim and rough, ice being deposited on their surfaces, apparently in a crust. When the temperature was raised so as to melt the ice and evaporate the resulting drops, the surfaces of the salt crystals appeared rough as if they had been etched by the super-cooled water before freezing occurred.

This work is of particular meteorological importance in view of the fact that sodium chloride nuclei derived from sea spray are abundant in the atmosphere. Ample proof of this has been provided by Dessens (1947) who has caught condensation nuclei on gossamer threads and examined them microscopically. In country air fairly remote from industrial areas he has identified the great majority of nuclei, greater than $10^{-5}$ cm in diameter, as consisting of sodium chloride crystals.

## 3. Discussion of experimental results

We now proceed to discuss the implications of the results of the foregoing experiments, with particular reference to the physical nature of the freezing nuclei.

### (a) *Nuclei operative between 0°C and —32°C*

These are the Findeisen " Type 1 " nuclei, most of which produce ice crystals at water saturation, although a few apparently become active at slightly lower vapour pressures, *i.e.*, at high ice-supersaturations. This immediately suggests that they cannot be true sublimation nuclei which should become effective at, or very near to ice-saturation.

Rau's observations that, in this temperature range, ice formation started at definite spots in the liquid, suggest that the responsible nuclei are solid and insoluble, a conclusion supported by Meyer and Pfaff (1935) who were able to remove them

by filtration and so super-cool water to —33°C. Weickmann's identification of small solid particles in melted cirrus and the fact that the areas where these particles were deposited on a microscope slide coincided with the areas of ice crystal growth on subsequent cooling, also indicate that solid particles are mainly responsible for initiating ice formation in droplets at these temperatures.

The efficiency of these "freezing nuclei" will depend on their size, shape, crystal structure and surface topography. According to Thomson (1939), the degree of super-cooling which can be attained is inversely proportional to the radius of the responsible nucleus, assumed isomorphous with ice. Although we are not generally concerned with isomorphic nuclei, presumably the maximum super-cooling will, in general, decrease with increase of nuclear size. This is confirmed by Meyer and Pfaff being able to prolong super-cooling to —33°C by repeated distillation and filtration, in which particles of diameter greater than $10^{-6}$ cm were removed.

We are now led to a possible explanation of the sterilisation of a nucleus by several alternate freezings and thawings, and its replacement by another nucleus at a lower temperature, as observed by Rau. Freezing will first occur on the most suitable nucleus which in general will be the largest. On subsequent freezing and melting of the drop it is conceivable that this nucleus will disintegrate into several smaller fragments and so become sterile. Freezing will then occur on the next most favourable nucleus which will be disrupted in turn, this process continuing until the nuclei cannot be further subdivided. At this stage, maximum super-cooling will be attained. This explanation receives support from electron-micrographs of aerosol, which consists largely of loosely bound aggregates of solid particles, the smallest of which are about $10^{-6}$ cm in diameter. It can be imagined that such particles are torn apart when surrounded by water which is alternately frozen and melted several times.

Further confirmation comes from the work of Dorsey (1938), who found that the freezing point of water could be influenced materially both by heating and letting it stand. Previous heating to 95°C caused depression of the freezing point. On taking a sample from the bottom of the vessel which had stood for two weeks, Dorsey found that the freezing point had risen by 5°C, but it was restored to its original value by alternate freezing and melting of the water. Again, these results can be explained if the freezing nuclei are assumed to consist of fairly loose aggregates of smaller particles. Heating might cause disintegration of the aggregates and consequent lowering of the freezing point. On the other hand, the larger particles would settle towards the bottom of the vessel during standing, where they might coagulate and cause elevation of the freezing point. Disruption of these particles by alternate freezing and thawing would enable the freezing point to return to its original value.

Rau's observations reveal another factor determining the efficiency of a freezing nucleus, its enhancement by drying-off of the surfaces. Ludlam (1948) has suggested that certain nuclei act by allowing a water skin to freeze and so produce an ice nucleus which will then continue to grow by sublimation, and has identified them

with the Findeisen " Type 1 " nuclei.

A solid nucleus which becomes dried-off at the surface or edge of a drop will retain an adsorbed film of water vapour of much higher concentration than the surrounding vapour. This sub-microscopic film is known as the Volmer layer, the molecules of which appear to move along the surface of the substrate until they find a place where they can fit into the crystal structure, and show a particular tendency to line up along steps or cracks in the surface. It may be that the oriented growth of ice crystals can occur on the surfaces of some solid nuclei in such a manner. This process will be much facilitated of course, if the atomic pattern of the substrate is close to that of ice, equality of lattice spacing being, in any case, important. Mobility of the Volmer layer plays an important part in crystal growth, the reduction of which, Dobson (1949) has invoked to explain the appearance of " glassy ice " at temperatures below —90°C on the thimble of a frost-point hygrometer.

The fact that a desiccated nucleus is more efficient in causing freezing of a water drop then when completely wetted, suggests that, in the former case, freezing occurs before the Volmer layer has time to reach equilibrium with the freshly condensed liquid. Once equilibrium is reached, the efficiency of the Volmer layer appears to be destroyed.

The reduced efficiency of nuclei subjected to more rapidly increasing relative humidities (Findeisen), is probably another manifestation of the same property.

We may now conceive of a second factor contributing to the sterilisation of nuclei after alternate freezings and thawings of a droplet. A " dried-off " nucleus situated on the surface of a drop and causing it to freeze at a relatively high temperature, may, after several such freezings and meltings, find itself in the interior of the droplet where, after becoming completely wetted, it will promote freezing only at a lower temperature.

Pits and cracks in the surface of a nucleus may enable bulk liquid water to appear at humidities well below water-saturation, by capillary condensation. This might account for the small number of ice crystals appearing at high ice-supersaturations reported by Weickmann, Findeisen and Palmer (1949).

### (b) *Nuclei operative between —32°C and —41°C*

The demonstrations by Rau, and Meyer and Pfaff that water cannot be supercooled below —33°C in the presence of solid, insoluble nuclei, and the sudden increase in the number of ice particles occurring in expansion chamber experiments at this temperature, indicate the introduction of a new type of nucleus, most probably a soluble one. Further support is provided by the fact that nuclei operating below —32°C react to changes in expansion rate in the reverse manner to the Findeisen " Type 1 " nuclei acting at higher temperatures.

Findeisen's suggestion of a second type of sublimation nucleus becoming operative at about —32°C. is denied by Fournier d'Albe's (1949) observations that these nuclei are effective only below the dew-point, which strongly suggests that they produce ice by the freezing of droplets.

Hollstein's work on the freezing of common salt solutions, and her observations that ice forms on sodium chloride crystals below about —30°C, together with the fact that particles of sea salt are abundant in the atmosphere, suggests that the new nuclei may consist of sea salt. Below about 70 per cent relative humidity they will tend to exist as crystals and at higher humidities as droplets of solution. At sufficiently low temperatures, both ice and salt will crystallise out from the droplets provided that there are particles in the droplets on which the salt and ice crystals can form. Complete solidification is usually assumed to occur at the eutectic point, but as Hollstein has shown, it is more likely to occur at lower temperatures (about —35°C), depending on the efficiency of the ice and salt forming nuclei. This would account for the critical temperature not being quite definite as reported by Fournier d'Albe. If the salt were homogeneous and chemically pure, and the condensed water entirely free of foreign particles, freezing would presumably be delayed as suggested earlier. Fournier has confirmed this by showing that chemically pure NaCl and $NaNO_3$ can form ice crystals in clean air only below —41°C, this limit being definite within 0·2°C.

Contaminated salt crystals suddenly subjected to increasing relative humidity at low temperatures, (as in Hollsteins' experiments), may not dissolve completely because of their diminished solubility, but will form a water skin in virtue of their hygroscopicity, this behaving as a saturated solution of the salt from which ice will separate at the " effective " eutectic temperature.

We have still to explain why the Findeisen " Type 2 " nuclei, now assumed to consist of contaminated droplets of sea salt solution produce ice crystals at higher temperatures at greater expansion rates, remembering that the dependence is upon the rate of expansion prevailing at the condensation point. At high rates of expansion, and so high rates of cooling, the salt may not be able to diffuse quickly enough to keep pace with the increasing growth of the droplet, causing a concentration gradient to be set up inside it. *The outer layers of the drop will therefore be relatively dilute, and separation of ice will occur on the outside of the drop at a higher temperature* as shown by Fig. 1. This process would require of course, the presence of solid particles to act as nuclei. According to Weickmann such particles always tend to move towards the outer surface of the drop and, in any case, may be deposited there from the air. Fournier observed that crystallisation apparently occurred during growth of the drop, there being no increase in the number of ice crystals after the expansion was completed, which is consistent with the suggestion that the freezing begins before uniform concentration of salt is attained. Once the condensation point is reached the drop will grow quickly at first, but thereafter more slowly as dilution of the outer layers will reduce the rate of condensation at the surface. Thus, the rate of expansion existing at the condensation point is the controlling factor, that prevailing after the drop has grown to almost full size having little effect.

It is suggested then, that the nuclei initiating ice crystal formation in the temperature range —32°C to —41°C consist of sea salt contaminated by foreign particles. Dessens (1949) states however, that the majority of sea salt nuclei which

he has examined consist of pure sodium chloride crystals and does not mention the appearance of contaminating particles. As the concentration of —32°C nuclei is only about 1 per $cm^3$ compared with 200 per $cm^3$ for the total number of sea salt nuclei, this is perhaps not surprising ; in any case, such particles would probably escape detection at the magnifications employed. According to Fournier, pure NaCl crystals are ineffective until —41°C which may partly account for the relative abundance of nuclei operative below this temperature, although other types of nuclei may also exist in this class (see next section).

These assumptions also afford an explanation of the observations that the —32°C nuclei are relatively less abundant over the sea, at high levels, and on the coast with a sea breeze (Cwilong), and above inversions and the haze top (Palmer). In these localities and under these conditions, the sea salt nuclei will have a smaller chance of becoming contaminated by foreign particles from land sources, and so will be more likely to operate below —41°C than above.

Reference may also be made here to the water fogs observed by Simpson below —30°C in the Antarctic. The most common condensation nuclei in this region are sea salt which, as we have seen, can remain liquid down to these temperatures.

### (c) *Nuclei operative below* —41°C

Fournier has shown that in clean air droplets of *pure* salt solutions freeze at a definite temperature of just below —41°C, this critical temperature apparently being independent of the nature of the salt and the degree of super-saturation. Dobson (1949) has inferred from this that the critical temperature of —41°C is a property of water and not of the nucleus. This point of view is not compatible, however, with the work of Regener and Rau and the theoretical conclusions of Krastanow, according to which the spontaneous formation of ice crystals from liquid water cannot occur at this temperature, while any other mechanism implies the intervention of some kind of nucleus. Nevertheless, this freezing temperature does seem to be independent of the crystal type in its solid state. The possibility arises, however, that the behaviour of the water may be modified by the solution of the crystal. When an electrolytically active salt dissolves in water, dissociation occurs and the ions so formed become surrounded by a group of neutral water molecules, the group moving as a whole under the influence of an applied electric field. It may be that such ionic groups can act as nuclei for ice formation. According to J. J. Thomson (1939), the temperature $T_r$ at which an isomorphic particle of equivalent radius $r$ can initiate freezing is given by the relation :—

$$\frac{T - T_r}{T} = \frac{2\sigma_{SL}M}{r\,Q\,d}$$

where $T$ is the " normal " freezing temperature (with isomorphic particle of infinite radius), $\sigma_{SL}$ is the interfacial surface energy at the particle/liquid boundary, $M$ the molecular weight and $d$ the density of the solid phase, and $Q$ the molecular heat of fusion. Of these quantities, $\sigma_{SL}$ is unknown. Volmer (1939, p. 181) suggests,

however, that a reasonable estimate of its value may be obtained by multiplying the free surface energy of the liquid phase, $\sigma_{LV}$, by the ratio of the latent heat of fusion, $L_f$, to the latent heat of vaporisation, $L_V$, *i.e.*, $\sigma_{SL} = \sigma_{LV}\ L_f/L_V$. Calculated in this manner, the mean value of $\sigma_{SL}$ over the temperature range 0°C to —60°C is about 10·5 ergs cm$^{-2}$. Putting $T = 273$°A, $M = 18$, $d = 0{\cdot}92$ gm cm$^{-3}$, $Q = 6 \times 10^{10}$ ergs/mol., we have :—

$$T - T_r = \Delta T = \frac{-1{\cdot}87 \times 10^{-6}}{r}\ ^{\circ}\mathrm{C}$$

Thus the maximum degree of super-cooling $\Delta T$, is inversely proportional to the radius of the isomorphic nucleus. Corresponding values of $\Delta T$ and $r$ are given in the following table.

| $\Delta T$ | $r$ |
|---|---|
| —0·2°C | $9{\cdot}3 \times 10^{-7}$ cm |
| —10°C | $1{\cdot}87 \times 10^{-7}$ cm |
| —33°C | $5{\cdot}66 \times 10^{-8}$ cm |
| —41°C | $4{\cdot}56 \times 10^{-8}$ cm |
| —68°C | $2{\cdot}75 \times 10^{-8}$ cm |

It appears then, that isomorphic nuclei of radius greater than about $2 \times 10^{-7}$ cm must be removed if water is to be super-cooled below —10°C, but of course, it would require considerably larger non-isomorphic nuclei to initiate freezing at the same temperature. An isomorphic nucleus of radius only $2{\cdot}75 \times 10^{-8}$ cm would, according to the above theory, initiate freezing at —68°C ; an aggregate of only five molecules arranged tetrahedrally as in the ice lattice, is required to produce a nucleus of this effective radius. A sphere of radius $4{\cdot}56 \times 10^{-8}$ cm centred on a given molecule in the ice lattice would contain eighteen molecules, hence an aggregate of about this number would be required to promote freezing at —41°C. The relative probability of spontaneous formation of aggregates of different sizes in homogeneous liquid water, and the relative time intervals required, cannot be calculated readily as X-ray analysis indicates that molecular groups of various sizes normally exist as a consequence of the polar properties of the molecules. It is fairly safe to assume, however, that the probability will decrease and the time required will increase rapidly with the total number of molecules required to form the initial ice-bud, and that we can expect such " chance aggregates " to form only at low temperatures. Thus, while an aggregate of five molecules may arise spontaneously at —68°C, the formation of an aggregate of eighteen at —41°C would be a much more improbable event, especially in view of the fact that the ice-bud must be an ordered structure requiring a peculiar orientation of the molecules. If ions are present in the water however, aggregates of water molecules are formed around the ions as already stated, and the experimental determination of their radius in dilute solutions of electrolytes reveals that they are of about the right size to produce freezing at —41°C according to the above theory.

Such a process would not necessarily be restricted to droplets of sodium chloride solution. Droplets of dilute sulphuric acid formed from industrially produced $SO_2$,

and of nitric acid formed from nitrous-oxide are also present in the atmosphere, and may act in a similar manner. Other very soluble nuclei, formed in the upper atmosphere by the action of ultra-violet radiation, *e.g.*, $O_3$, $H_2O_2$, $NH_3$, may also play their part as indicated by the recent observations of Brewer and Palmer, who found that nuclei operative at about —41°C were produced when a chamber containing nitrates was irradiated by ultra-violet light.

An interesting sidelight is thrown on these suggestions by a recent report by Oliver and Oliver (1949) on the incidence of ice-fogs at low temperatures in central Alaska. Persistent ice-fogs form only when the temperature falls below about —41°C, and only in regions with an abundance of smoke particles. The authors suggest that the ice-fogs form on smoke nuclei, the process being aided by the extra moisture liberated in combustion. It may be that the nuclei actually consisted of diluted sulphuric acid as suggested above. It will be recalled that Weickmann found similarly produced nuclei to be ineffective in the formation of ice crystals at temperatures down to about —39°C.

### (d) *The behaviour of AgI and $CdI_2$ nuclei*

The mode of action of these crystals is of special interest as they can apparently produce ice crystals at higher temperatures than most nuclei occurring naturally in the atmosphere.

Silver iodide has been reported active at temperatures as high as —4°C by Vonnegut (1947) and —7°C by Fournier (1949). The latter observed that ice crystals appeared only at the dew-point and so could not have been formed by direct sublimation. It appears then, that AgI crystals act as particularly efficient, insoluble, freezing nuclei in virtue of their crystal structure being fairly close to that of ice.

Cadmium iodide, in contrast to silver iodide, is soluble and slightly hygroscopic. On spraying droplets of dilute solution of the pure salt into his chamber containing clean air, Fournier found that, during the first expansion, ice crystals appeared only below —41°C and at the dew point. Under these conditions, the nucleus, presumably being a droplet of $CdI_2$ solution, behaved exactly as the nuclei of all other soluble salts which were tested. When the ice particle was allowed to evaporate however, it left behind a nucleus on which ice would grow again after very small expansions producing temperatures only very slightly below the frost-point. Fournier concluded from this, that true sublimation nuclei had been formed. If the nuclei were warmed above —9·1°C however, they ceased to act in this manner, subsequent large expansions producing only water droplets although the final temperature fell to —13·5°C.

It is suggested that warming to temperatures below —9·1°C caused only partial evaporation of the ice particles and that a microscopic oriented film of ice was retained on the surface of the $CdI_2$ crystals. According to Frank and van der Merwe (1949), such a layer may be deformed into exact fit with the substrate if the degree of misfit between the crystal lattices of the two substances is less than a certain critical amount. A nucleus so formed, will now act as a true sublimation nucleus and grow rapidly under the increasing relative humidities produced by

subsequent cooling of the chamber. It is however, the ice film which is instrumental in such a process, the $CdI_2$ crystal acting only indirectly by providing a surface on which a strongly bound layer of ice can form. At temperatures above —9·1°C it is conceivable that this bonding of the ice to the substrate is weakened and the ice film loses its crystal structure, and will now behave as a liquid film. If this is the case, the critical temperature of —9·1°C should not be far removed from the eutectic temperature of $CdI_2$ solution, which is the temperature at which ice and the salt crystallise out entirely in solid form ; (according to the *International Critical Tables*, this is —8·5°C). The liquid film will increase in thickness in virtue of the hygroscopicity of the crystal and in effect a droplet of saturated solution of $CdI_2$ will be formed, which will grow quickly on subsequent expansion of the air in the chamber.

(e) *The spontaneous formation of ice crystals*

The Volmer-Krastanow theory leads us to believe that, at sufficiently low temperatures and high super-saturations, ice crystals may form spontaneously, without the aid of foreign nuclei, from homogeneous water vapour, and, at slightly higher temperatures, from pure super-cooled water. Krastanow calculates that such ice formation from the vapour phase will occur below —65°C and although, as already pointed out, the numerical values are doubtful, there is some experimental evidence that they provide a good estimate of the transition temperatures.

Using filtered air, Regener (1941) observed water-fogs in his chamber at —50°C. With expansion ratios of 1·6 to 1·8 depending on the initial temperature (which varied between 0°C and —40°C), he produced fine ice mists and no water droplets. Although he does not state the instantaneous temperature reached at the end of these large expansions, they would almost certainly penetrate at least into the region required by Krastanow's theory.

Rau (1944), in the absence of freezing nuclei, claims that he was able to maintain water drops in the liquid state down to —70°C for several hours. At —72°C however, the droplets froze uniformly without the selection of special spots, the growing crystals being mainly octahedral in shape, with occasional tetrahedra and cubes. This observation is of interest in connection with the suggestion made by Frank (1946) that the molecular configuration of water changes at such low temperatures. He postulates that complex molecules may arise, the simplest form consisting of six $H_2O$ molecules with the O atoms arranged at the corners of a regular octahedron, and that such a configuration would involve a reduction in the surface tension of the water and so account for the flattening of the droplets observed by Rau to occur below —55°C. Below —72°C, Rau found no water droplets occurring on his condensation surface but a fine ice mist appeared, presumably arising by sublimation from the vapour phase. These crystals also contained octahedrons and tetrahedrons.

However, some doubt has been thrown on Rau's work by Cwilong (1947 c) who failed to reproduce the former's results unless a small amount of ether or acetone vapour was introduced into the chamber, whereupon all of them were

obtained with the one difference that the spontaneous freezing point occurred at about —90°C instead of —70°C as observed by Rau. Cwilong suggests that Rau's chamber may have been contaminated by evaporation of the lubricant from the fan used to circulate the air. Whether sufficient concentration of contaminating vapour could be produced in this way to influence the freezing point appreciably, is doubtful. In this connection it is interesting that Weickmann (1947) quotes Rau as stating that "the penetration of alcohol vapour from the cooling system into the experimental chamber was impossible." The effect of small concentrations of foreign vapours on the freezing temperatures and the crystal habit requires further investigation.

Finally we shall refer to the work of Sander and Damköhler (1943) who have produced a slightly refined version of Krastanow's theory of spontaneous bud formation, and have tested it by measuring the supersaturation required to produce droplets or crystals at a given rate (1 $cm^{-3}$ $sec^{-1}$) at different temperatures. At temperatures above —62°C, when droplets are formed, they find a linear relationship between the logarithm of the required supersaturation and the reciprocal of the temperature. In order to obtain agreement with their theoretical curve, Sander and Damköhler assumed that the curve of surface tension of super-cooled water plotted against temperature took a certain form, passing through a maximum at —50°C and then falling at lower temperatures. In this connection, we may recall Rau's observations that water droplets begin to flatten at about the same temperature. At about —62°C ice crystals appeared spontaneously from the vapour, the exact temperature at which they formed being difficult to estimate. At this temperature however, a sharp discontinuity occurred in the experimental curve, indicating that below —62°C a type of bud was arising which required higher water vapour pressures for its formation than the type of bud at higher temperatures. This state of affairs is in complete disagreement with the underlying assumptions of the Volmer-Krastanow theory and indicates that such a theory may not be valid at these temperatures. Sander and Damköhler, seeking an explanation for the breakdown of the theory, suggest that at such low temperatures, the surface molecules of a bud will lose their mobility and so only ice buds are formed irrespective of the existing supersaturation.

## 4. Conclusions

From a careful study of the existing experimental evidence together with straightforward physical reasoning, it appears possible to arrive at some reasonable conclusions concerning the nature of ice-forming nuclei and their mode of action. The theories advanced in this paper appear to unify many of the results which have emerged from recent experiments but, of course, they may have to be revised in the light of future investigations.

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## References


Becker, R. and Döring, W. 1935 *Ann. Phys.*, Leipzig, **24**, p. 749.
Dessens, H. 1947 *Météorologie*, Paris, p. 321.
,, 1949 *Quart. J. R. Met. Soc.*, London, **75**, p. 23.
Dobson, G. M. B. 1949 *Ibid.*, **75**, p. 117.
Dorsey, N. E. 1938 *J. Res. Nat. Bureau Standards*, Washington, **20**, p. 799.
Cwilong, B. M. 1947a *Proc. Roy. Soc.*, London, A, **190**, p. 137.
1947b *Nature*, London, **160**, p. 198.
1947c *J. Glaciology*, **2**, p. 53.
1949 *Nature*, London, **163**, p. 727.
Findeisen, W. 1938 *Met. Zeit.*, Braunschweig, **55**, p. 121.
Findeisen, W. and Schulz, G. 1944 *Forsch. u. Erfahrb. des Reichs. Wetterdnst.*, Berlin, Ser. A, No. 27.
Fournier d'Albe, E. M. 1949 *Quart. J. R. Met. Soc.*, London, **75**, p. 1.
Frank, F. C. 1946 *Nature*, London, **157**, p. 276.
Frank, F. C. and van der Merwe, J. H. 1949 *Proc. Roy. Soc.*, London, A, **198**, p. 216.
Hollstein, Frl. 1947 *Die Eisphase in der Atmosphäre, Rep. und Trans.*, No. 716, M.O.S. (A), Volkenrode.
Krastanow, L. 1940 *Met. Zeit.*, Braunschweig, **57**, p. 357.
,, 1941 *Ibid.*, **58**, p. 37.
Ludlam, F. H. 1948 *Quart. J. R. Met. Soc.*, London, **74**, p. 39.
Meyer, J. and Pfaff, W. 1935 *Zeit. f. Anorg. Chemie.*, **224**, p. 305.
Oliver, V. J. and Oliver, M. B. 1949 *Bull. Amer. Met. Soc.*, Boston, Mass., **30**, p. 15.
Palmer, H. P. 1949 *Quart. J. R. Met. Soc.*, London, **75**, p. 15.
Rau, W. 1944 *Schr. Deutsch. Akad. Luft.*, Berlin, **8**, p. 65.
Regener, E. 1941 *Ibid.*, H. **37**, p. 15.
Sander, A. and Damköhler, G. 1943 *Die Naturwiss.*, **31**, p. 460.
Simpson, G. C. 1941 *Quart. J. R. Met. Soc.*, London, **67**, p. 99.
Thomson, J. J. 1939 *In Kinetik der Phasenbildung* by Volmer, Dresden (Steinkoff), p. 17.
Volmer, M. 1939 *Ibid.*
Vonnegut, B. 1947 *J. App. Phys.*, Lancaster, Pa., **18**, p. 593.
Wegener, A. 1911 *Thermodynamik der Atmosphäre*, Leipzig, (J. A. Barth).
Weickmann, H. 1942 *Forsch. Bericht.*, 1730, Deutsch. Luft., Berlin.
,, 1947 *Die Eisphase in der Atmosphäre, Rep. und Trans.* No. 716, M.O.S. (A) Volkenrode.

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# Constant-Height Balloons — Calculation of Geostrophic Departures

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SUMMARY

The accelerations observed during the flight of seven constant-height balloons are examined.

These observations extended over periods of 2 to 4½ hr. It is found that the values of the geostrophic departures determined from the accelerations are closely comparable with the values deduced independently from contour charts and observed winds.

The variations in the acceleration are greater than would be expected from errors in wind determination alone ; it is deduced that this is due to the wind at the level of flight being affected by eddies giving rise to fluctuations of the order of 3 to 4 kt in the value of the mean wind determined over half-hour intervals.

## 1. HISTORICAL

During a discussion between Asst. Professor P. A. Sheppard and Mr. C. S. Durst it was suggested that useful insight into the magnitude of the terms in the fundamental hydrodynamical equations might be obtained from the measurement of wind by balloons flying at a constant level. As a result the flights discussed in the following notes were made during April, 1943 to March, 1944.

The organisation of these radio soundings was put in the hands of Dr. D. N. Harrison. The making of the observations involved a considerable measure of dislocation in the routine upper air observations over the British Isles as staff had to be specially brought on to man the stations.

## 2. THEORY

### (a) *The acceleration of air deduced from routine observations*

The equation expressing the total derivative of the horizontal component **V** of the wind in terms of the partial derivatives is :—

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $u$, $v$ are the components of **V** in the $x$ and $y$ directions and $w$ is the vertical component of the wind.

An approximation to the average value of the local horizontal acceleration $\frac{\partial \mathbf{V}}{\partial t}$ over a period of some hours may be obtained from the variation of wind given by consecutive routine soundings from a single station, and an estimate of the value of the terms $u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$ from a comparison of simultaneous routine soundings from a network of stations. The term $w\frac{\partial \mathbf{V}}{\partial z}$ may be calculated by evaluating $w$ and $\frac{\partial \mathbf{V}}{\partial z}$ independently, the former by the method developed by Hewson (1936), Graham (1947) and others, and the latter from the variation of wind with height in routine soundings.

(b) *The acceleration of air deduced from constant height balloons*

The total horizontal acceleration $\frac{d\mathbf{V}}{dt}$ is most readily obtained from observed accelerations of a balloon flying at a constant height. The observed accelerations when smoothed give the value of the term $\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$ directly. It is still necessary to evaluate the term $w\frac{\partial \mathbf{V}}{\partial z}$ by the method indicated in the previous paragraph. A simplification arises in the calculation of $w$, however, in that it is no longer necessary to determine the trajectory of the air as this is given by the path of the "constant height" balloon.

The frictional terms in the general equation of motion are usually assumed to be negligible in comparison with the other terms so that the equation may be reduced to (vide Petterssen 1940)

$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho}\nabla p + \mathbf{V} \times \mathbf{k}\, l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $\frac{d\mathbf{V}}{dt}$ is the component of the acceleration, $\mathbf{k}$ is the unit vertical vector, $\rho$ density, $p$ pressure and $l$ the coriolis parameter.

This equation may be written as

$$\mathbf{V} - \mathbf{V}_G = \frac{1}{l}\,\mathbf{k} \times \frac{d\mathbf{V}}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

if $\mathbf{V}_G$ is the geostrophic wind.

Thus, the magnitude of the geostrophic departure $\mathbf{V} - \mathbf{V}_G$ is proportional to the magnitude of $\frac{d\mathbf{V}}{dt}$, and the direction of $\mathbf{V} - \mathbf{V}_G$ is perpendicular to that of $\frac{d\mathbf{V}}{dt}$ and directed to the left of it. When $\mathbf{V} - \mathbf{V}_G$ is expressed in kt and $\frac{d\mathbf{V}}{dt}$ in kt/hr, the ratio of their respective magnitudes is 2·4 : 1 (Latitude 52°).

In the present paper an attempt has been made to measure the value of $\frac{d\mathbf{V}}{dt}$ directly by following constant height balloons and thereby to derive values of $\mathbf{V} - \mathbf{V}_G$. Comparison is then made with values derived by other methods.

## 3. Data available

Seven radio soundings were made from April 1943 to March 1944, and the balloons were designed to follow a path along a predetermined pressure level, generally in the neighbourhood of 400 or 500 mb. On reaching the predetermined level further ascent and descent of the balloon was controlled at first by the escape of gas from the balloon and the jettisoning at intervals of small sand bags. In later soundings the automatic jettisoning of fluid took place when the balloon fell below the predetermined pressure level, and a more constant level flight was maintained. The balloons were first followed by the stations from which they were released and later as they got out of range by other stations. The period of flight for which data are available varied from 2 to 4½ hr.

Since information on the accuracy of the fixes of the positions of balloons was very necessary, consideration was given to the " Class " of the intersection of the rays from the triangle of radio wind finding stations. For each fix the observer put down one of four Class letters, A, B, C or D which were supposed to represent probable vector errors of wind measurement of between 0 and $2\frac{3}{4}$ kt, $2\frac{3}{4}$ and $5\frac{1}{2}$ kt, $5\frac{1}{2}$ and 11 kt, and over 11 kt respectively. In the result, however, Sheppard in a manuscript report reached the conclusion that these errors were exceeded on only 25 per cent of occasions rather than on 50 per cent. This practice was a routine with radio wind observations, and though in the constant height balloons the observations were often at rather extreme ranges, there is no reason to believe that the accuracies shown by the class letters were not correct.

## 4. Treatment of data

### (a) *Total accelerations and geostrophic departures*

The average value of the acceleration $\frac{d\mathbf{V}}{dt}$ was calculated for each flight. $\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$ was first obtained by smoothing the observed mean half-hourly values of the wind by fitting them to a straight line by the method of least squares. The average value of $\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$ was then given by the shape of this line. $w$ was obtained by identifying the " balloon " at the end of its path by the temperature and the wet-bulb potential temperature of its environment, and then calculating the necessary amount of ascent or descent during flight to account for the observed changes (during flight) of these elements. $\partial \mathbf{V}/\partial z$ was obtained from the average height variation in the wind over a 200 mb range (centred at the level of flight) as given by the routine soundings at Larkhill before, during and immediately after each constant level balloon flight.

The value of the geostrophic departure $\mathbf{V} - \mathbf{V}_G$ was then calculated from $\frac{d\mathbf{V}}{dt}$ by means of Eq. (3) and its value compared with that deduced by other methods.

### (b) *The error introduced by change in height of balloon*

The variation in height of a balloon flying with the wind along a constant pressure surface is small, and the apparent horizontal acceleration of the balloon due to this cause may be neglected. During all of the flights under discussion, however, the balloon departed from its predetermined pressure level. These departures were considerable and the observed mean half-hourly winds were therefore first corrected for these departures using the value of $\frac{\partial \mathbf{V}}{\partial z}$ obtained in the calculation of $w\frac{\partial \mathbf{V}}{\partial z}$ above. During the third flight the balloon actually passed through a frontal surface ; the data for this flight were discarded, though the observations are included in Appendix I, on the ground that the accelerations would not be comparable with those deduced by other means.

(c) *The fluctuations of wind due to eddies*

On the assumption that the value obtained for $\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$ approximated to the true mean of the various observed accelerations, the mean square error of a single observed acceleration was calculated for each flight. Departures of the observed wind from its smoothed value were also considered and the magnitude of the departure due to eddies alone was calculated.

## 5. Results

(a) *Total acceleration and geostrophic departures*

In Table I are set out the following for each flight :—

(i) the value of the total acceleration $\frac{d\mathbf{V}}{dt}$ from the constant height balloons (column L) and the theoretical geostrophic departure $\mathbf{V} - \mathbf{V}_G$ calculated from it (column M) ;

(ii) the value of the geostrophic departure at the mid-point of flight (column H) deduced from the comparison of the mean wind as obtained from the constant height balloon (column E) and the geostrophic winds measured off the routine working contour charts (columns F and G) ;

(iii) the values of the geostrophic departures at 12.00 hr deduced from the comparison of the routine observations of upper wind made at Larkhill, Downham Market and Fazakerley and the pressure gradients shown on contour charts drawn specially for the purpose of this investigation (column I and J) (Two workers, Gilbert and Sawyer, made charts based on pressures only without reference to any upper wind data at all and from these charts measured the pressure gradients. Each made charts quite independently of the other so that columns I and J are unprejudiced) ;

(iv) the value of the geostrophic departure deduced from an independent estimate of the acceleration (column K) ; this method of estimating geostrophic departures was suggested by Durst (1940) and has since been modified by Sawyer. It consisted in constructing the trajectory of the air at the 500-mb level on the basis of geostrophic motion, and estimating the wind at various intervals along it. By smoothing the irregularities in velocity and direction, estimates were obtained of the accelerations over a period of 36 hr, from which the geostrophic departures shown in column K were deduced.

It should be noted that no account is taken of the term $w\frac{\partial \mathbf{V}}{\partial x}$ in the determination of the geostrophic departure in column K. A column M′ is therefore added for comparison giving the theoretical geostrophic departure calculated from $\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$. In Fig. 1 vectors are shown representing the geostrophic departures deduced from the accelerations of the constant-height balloons (column M) and the calculations made by Gilbert and Sawyer (columns I and J). There

TABLE I. COMPARISON OF GEOSTROPHIC DEPARTURES DEDUCED IN VARIOUS WAYS

| Sounding | Date | Time at mid-point of flight (G.M.T.) | Average pressure | Average wind (kt) | Geostrophic wind (kt) before flight | | Geostrophic wind (kt) during flight | | Geostrophic departure (kt) calculated from E, F and G | Geostrophic departure (kt) at 12.00 hr (first observer) | Geostrophic departure (kt) at 12.00 hr (second observer) | Geostrophic departure (kt) deduced from trajectories based in 500 mb contours | Average total acceleration (kt/hr) | Theoretical geostrophic departure calculated from L | Theoretical value of geostrophic departure calculated from $\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Time | Dir. Speed | Time | Dir. Speed | | | | | | | |
| A | B | C | D | E | F | | G | | H | I | J | K | L | M | M′ |
| 1 | 27. 4.43 | 18.00 | 471 | 283° 43 | 12.00 | 290° 50 | 18.00 | 300° 40 | 215° 12·7 | 143° 6·6 | 143° 25·9 | 090° 3·4 | 250° 4·6 | 160° 11·1 | 145° 8·2 |
| 2 | 28. 4.43 | 17.31 | 463 | 305° 43 | 12.00 | 210° 40 | 18.00 | 320° 30 | 276° 15·1 | 148° 5·4 | 120° 8·9 | 225° 0·7 | 200° 2·8 | 110° 6·7 | 081° 10·7 |
| 4 | 24.12.43 | 17.00 | 466 | 011° 56 | 12.00 | 020° 45 | 18.00 | 010° 40 | 009° 15·5 | 018° 15·5 | 022° 12·7 | 006° 5·3 | 070° 7·2 | 340° 17·4 | 354° 14·4 |
| 5 | 7. 1.44 | 17.58 | 530 | 287° 39 | 12.00 | 300° 45 | 18.00 | 300° 50 | 156° 14·8 | 212° 7·9 | 156° 36·4 | 333° 3·1 | 302° 3·3 | 212° 7·9 | 212° 7·9 |
| 6 | 18. 2.44 | 17.00 | 480 | 017° 63 | 12.00 | 030° 40 | 18.00 | 030° 50 | 342° 19·3 | 200° 1·0 | 309° 4·0 | 207° 4·1 | 166° 1·1 | 076° 2·5 | 065° 0·9 |
| 7 | 27. 3.44 | 17.15 | 371 | 308° 58 | 12.00 | 280° 50 | 18.00 | 330° 50 | 259° 18·0 | 238° 18·3 | 243° 27·1 | 330° 7·3 | 057° 6·3 | 327° 15·1 | 323° 14·4 |

Vector correlation coefficient between I and M is ·53 (·82 if sounding 7 is omitted)
,, ,, ,, ,, J and M is ·48 (·59 ,, ,, ,, ,, )
,, ,, ,, ,, H and M is ·35 (·35 ,, ,, ,, ,, )
,, ,, ,, ,, M′ and K is ·64 (·43 ,, ,, ,, ,, )
,, ,, ,, ,, M and K is ·61 (·40 ,, ,, ,, ,, )

appears to be fairly close agreement between the three sets of vectors. The vector correlation coefficient between the values in column I and column M is 0·53 and between the values in columns J and M 0·48.

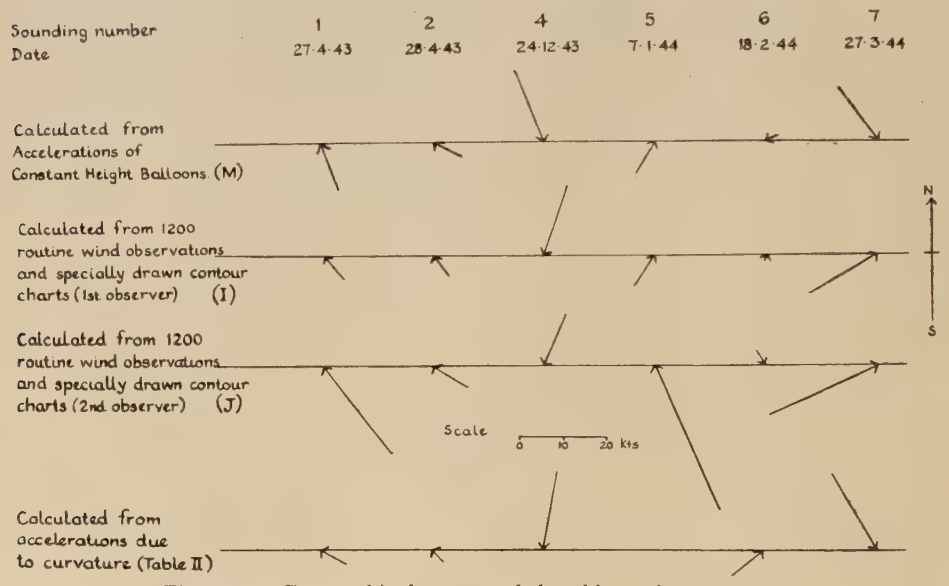



Figure 1. Geostrophic departures deduced by various means

The vector correlation coefficient between two sets of vector $\mathbf{V}_1$ . . . . . and $\mathbf{V}_2$ . . . . . is defined as :—

$$\frac{\Sigma \mid \mathbf{V}_1 \mid \mid \mathbf{V}_2 \mid \cos \theta_{12}}{\sqrt{(\Sigma \mathbf{V}_1^2 \Sigma \mathbf{V}_2^2)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $\theta_{12}$ is the angle between pairs of vectors $\mathbf{V}_1$ and $\mathbf{V}_2$. If the vectors are resolved into components $x_1$, $y_1$ and $x_2$, $y_2$, the coefficient is expressed as

$$\frac{\Sigma (x_1 x_2 + y_1 y_2)}{\sqrt{[\Sigma (x_1^2 + y_1^2) \Sigma (x_2^2 + y_2^2)]}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The vector correlations are not large in view of the few pairs of vector observations, but from expression (5) it is seen that a vector correlation coefficient can be looked on as consisting of twice the number of component pairs. It may be shown that the chance against getting such correlation coefficients from unrelated data is about 10 to 1. It is perhaps worth noting that if the last sounding had been omitted the correlations would have been 0·82 and 0·59 respectively and the chance against the correlation being accidental would have risen to a far higher figure.

The correlation coefficient between the geostrophic departures deduced from the working contour charts (column H of Table I) and column M is lower, 0·35. The correlation coefficient between columns M′ and K is ·64, and between columns M and K ·61, but the standard vector deviation of the figures in column K is small.

It is of interest to note that except for sounding No. 5 the theoretical geostrophic departures, column M, have a greater component in the direction of the geostrophic wind then at right angles to it. It may well be then that the departure of wind from geostrophic in the free air is due mainly to the curvature of the trajectory of the air.

Some calculations were made of the accelerations due to curvature in a manner set out in an unpublished memorandum by Gilbert (1945) which is based on a formula of Petterssen (1944) and from these geostrophic departures were calculated. These are set out in Table II.

TABLE II. GEOSTROPHIC DEPARTURES DEDUCED FROM ISOBARIC CURVATURE AND FROM THE TRAVEL OF CONSTANT-HEIGHT BALLOONS (° TRUE AND KT)

| | 1 | 2 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Deduced from isobaric curvature .. .. | 120° 7 | 140° 4 | 010° 18 | Small | 230° 9 | 330° 20 |
| Deduced from constant height balloons .. .. | 160° 11 | 110° 7 | 340° 17 | 212° 8 | 070° 3 | 327° 15 |

The vector correlation between these two sets of independent values is 0·87 and the chance that this is due to accident is very small. These values are also included in Fig. 1.

(b) *The error introduced by change in height of balloon and the acceleration due to vertical motion*

In Table III are set out the following for each flight :—

(i) The value of the total acceleration $d\mathbf{V}/dt$ from the constant height balloons (column L of Table I) ;

(ii) the value of the vertical component $w$ of the wind in mb/hr (column N) and the value of $w\ \partial\mathbf{V}/\partial z$ deduced from it (column P) ;

(iii) the average variation in the height of the balloon obtained by smoothing the observed mean half-hourly values by the method of least square (column Q) ;

(iv) the average value of the observed acceleration (obtained by smoothing the observed mean half-hourly wind by the method of least square without first correcting for change in height of balloon (column R).

TABLE III. VERTICAL VELOCITIES OF BALLOONS AND AVERAGE HEIGHT VARIATIONS

| Sounding | Average total acceleration kt/hr | $w$ mb/hr | $w\frac{\partial\mathbf{V}}{\partial z}$ kt/hr | Average height variation mb/hr | Average observed acceleration kt/hr | Root mean square error |
|---|---|---|---|---|---|---|
| | L | N | P | Q | R | |
| 1 | 250 4·6 | + 32 | 285 1·6 | + 33 | 248 4·4 | 2·1 |
| 2 | 200 2·8 | + 30 | 320 2·6 | + 37 | 215 2·5 | 1·6 |
| 4 | 070 7·2 | + 17 | 024 2·0 | + 4 | 091 6·3 | 2·6 |
| 5 | 302 3·3 | — 2 | < 0·1 | + 2 | 292 3·2 | 0·9 |
| 6 | 166 1·1 | — 12 | 172 0·7 | — 23 | 360 0·9 | 6·3 |
| 7 | 057 6·3 | — 4 | 115 0·4 | — 4 | 047 5·7 | 3·1 |

Positive sign in columns N and Q indicate an increase in height.

A striking feature of Table III is the agreement between columns N and Q. Indeed it would appear as if the mechanisms for maintaining the balloon in level flight were insensitive to height variations of the order observed. Thus any variation in the vertical component of the wind would be largely reflected in the variation in height of the balloon. Moreover, the average observed acceleration should approximate very closely to the value of the total acceleration $d\mathbf{V}/dt$. That this is so is borne out by the remarkable agreement between columns L and R. This agreement also suggests that the error introduced into the calculation of $d\mathbf{V}/dt$ by correcting for the variation in the height of the balloon is small and usually less than 0·2 or 0·3 kt/hr

The contribution of the term $w\ \partial\mathbf{V}/\partial z$ to the total acceleration was shown to be important by Durst and Sutcliffe (1938). A comparison of column L and column P confirms this.

In Table III there are given the root mean square errors involved in the calculation of the average observed acceleration. In the majority of cases these are less than the accelerations themselves. This confirms that the average observed accelerations and hence the total accelerations and the geostrophic departures calculated therefrom have a real meaning.

### (c) *Fluctuations of wind due to eddies*

In the course of the preparation of the figures of average observed acceleration (column R of Table III), accelerations $\Delta\mathbf{V}/\Delta T$ over a shorter interval of time were obtained from the mean winds over consecutive half-hour periods, and the mean square error of a single observation of $\Delta\mathbf{V}/\Delta T$ was found. This is tabulated below for each constant-level flight.

TABLE IV

| Sounding | Root mean square error of a single observation of $\Delta\mathbf{V}/\Delta T$ in kt/hr |
|---|---|
| 1 | 15·8 |
| 2 | 10·6 |
| 4 | 22·0 |
| 5 | 5·6 |
| 6 | 23·8 |
| 7 | 19·0 |

The mean square error of a single observation in $\Delta\mathbf{V}/\Delta T$ is greater than the magnitude of the average observed acceleration in all cases. Thus the acceleration obtained from the mean winds for two consecutive half-hour periods can give no reliable indication of the mean acceleration over a longer period. These values of $\Delta\mathbf{V}/\Delta T$ are greater than would be expected from errors due to the methods of wind determination alone : it is probable that this is due to the departure of the acceleration from its assumed constant value, *i.e.*, that there are fluctuations in acceleration due to eddies. It was supposed that some of the variations might be due to the change of wind with height and that change was allowed for by fitting a straight line to the observed mean winds and pressures. Even so the value of

the mean square error in $\Delta V/\Delta T$ was still of the same order of magnitude as given in Table IV above.

The departures of the observed values of the mean wind from the smoothed values may therefore be considered to be due to :

(i) errors in wind determination, and

(ii) departures of the acceleration from its assumed constant value due to eddies.

The magnitude of (i) can be calculated from the reported degree of accuracy* in the wind determination, and consequently the magnitude of the fluctuation in the mean wind due to eddies can be obtained (see Appendix II).

Table V gives the total mean square departure of the observed mean wind from its smoothed value, and also the mean square departures due to errors in wind determination and eddies separately. The total mean square departures were further analysed and the ratios of the components cross wind to the components along wind are given.

TABLE V. MEAN SQUARE DEPARTURE OF THE OBSERVED MEAN WIND (UNCORRECTED FOR CHANGE IN PRESSURE LEVEL) FROM ITS SMOOTHED VALUE (ROOT MEAN SQUARE DEPARTURE IN KNOTS)

| Sounding | Average pressure in mb | Total departure (cross wind/along wind component) | | Departure due to errors in wind determination | | Departure due to eddies | |
|---|---|---|---|---|---|---|---|
| | | | | (a) | (b) | (a) | (b) |
| 1 | 471 | 6·1 | ( ·76) | 2·8 | 3·9 | 5·9 | 4·7 |
| 2 | 463 | 4·5 | (1·08) | 2·8 | 3·9 | 3·5 | 2·2 |
| 4 | 466 | 8·0 | ( ·78) | 4·6 | 6·5 | 3·8 | small |
| 5 | 530 | 2·1 | (1·91) | 2·2 | 3·0 | small | small |
| 6 | 480 | 10·9 | ( ·44) | 4·8 | 6·7 | 9·8 | 8·6 |
| 7 | 371 | 7·0 | ( ·98) | 5·2 | 7·3 | 4·7 | small |
| Mean square value | | 6·6 | | 3·9 | 5·5 | | |

It will be seen from Table V that eddies produce fluctuations of the order of 3 to 4 kt in the mean wind determined over half hour periods.

Although the cross-wind component of the total departure of the observed wind from its smoothed value is in general less than the component along the wind it cannot be assumed that the fluctuations of wind due to eddies are greater along the wind than cross-wind. The effect may be due to the type of errors inherent in wind determination.

## 6. RELATED SYNOPTIC SITUATIONS

The synoptic situations relating to the various soundings are given in Table VI. Although data are insufficient to make broad generalizations it appears that fluctuations of wind due to eddies are greatest in Polar Continental air and in thundery conditions. This result is as might be expected.

* A value of the error of each radio wind observation is estimated by the observer ; these may be taken as a broad measure of the degree of accuracy. Columns (a) and (b) of Table V were calculated on the assumption that this reported degree of accuracy gave on the average the radius of the circle within which would be 75% and 50% respectively of the actual errors.

TABLE VI

| Sounding | Synoptic Situation or Air Mass | Curvature of 500-mb contours |
|---|---|---|
| 1 | Cold front. Thunderstorms | Slight |
| 2 | Warm frontal cloud | Slight |
| 4 | Occlusion to west | Markedly anticyclonic |
| 5 | Tropical maritime air | Slight |
| 6 | Polar continental air | Cyclonic |
| 7 | Tropical maritime air | Anticyclonic |

## 7. Conclusions

The general conclusions to be drawn from these observations are :

(i) By the method of following constant-height balloons values of the acceleration of the air are obtained which lead to estimates of the geostrophic departure of the correct magnitude.

(ii) Geostrophic departures can also be directly measured by comparing radio wind finding data with pressure contour charts with comparable accuracy.

(iii) Estimates of geostrophic departures obtained by constructing trajectories from the 500-mb contour do not appear to be accurate, possibly because the trajectories had to be smoothed over about 36 hr. This suggests that geostrophic departures are not controlled by the very large scale synoptic phenomena, *e.g.*, major depressions and anticyclones, but are more likely to be associated with irregularities of the synoptic pattern on a rather smaller scale.

(iv) The fairly close agreement between the average geostrophic departures calculated from constant-height balloons and the geostrophic departures obtained from a comparison of the pressure gradient and routine wind observation suggests that the use of the equation $\mathbf{V} - \mathbf{V}_G = \frac{1}{l}\,\mathbf{k} \times \frac{d\mathbf{V}}{dt}$ to determine geostrophic departure $\mathbf{V} - \mathbf{V}_G$ from the total acceleration $\frac{d\mathbf{V}}{dt}$ is justified, *i.e.*, frictional forces could, in these cases at any rate, be neglected in comparison with $\frac{d\mathbf{V}}{dt}$ and $\mathbf{V} - \mathbf{V}_G$.

(v) There is evidence that in the upper air there are eddies which cause fluctuations of wind of the order of 3 or 4 kt from one half hour to another.

## 8. Acknowledgment

Our thanks are due to the Director, Meteorological Office, for permission to publish this paper. We have also gratefully to acknowledge the assistance of Dr. Harrison in obtaining the data and of Mr. J. S. Sawyer in some of the discussion, as is mentioned in section 5.

## References


Durst, C. S. 1940 *Quart. J. Met. Soc.*, London, **64**, p. 171.
Durst, C. S. and Sutcliffe, R. C. 1938 *Ibid.*, **64**, pp. 75 and 240.
Gilbert, G. H. 1945 *S.D.T.M.*, Met. Off., London, No. 98.
Graham, R. C. 1947 *Quart. J. R. Met. Soc.*, London, **73**, p. 407.
Hewson, E. W. 1936 *Ibid.*, **62**, p. 387.
Petterssen, S. 1940 *Weather Analysis and Forecasting*, New York (McGraw Hill), p. 210.
Petterssen, S. 1944 *S.D.T.M.*, Met. Off., London, No. 78.


## Appendix I

## AVAILABLE DATA

Sounding 1 — 27 April 1943 Fazakerley release 1534 G.M.T.

| G.M.T. | 1600* | 1630 | 1700 | 1730 | 1800 | 1830 | 1900 | 1930 | 2000 |
|---|---|---|---|---|---|---|---|---|---|
| Direction | 288 | 288 | 287 | 278 | 285 | 283 | 280 | 280 | |
| Speed km/hr | 86 | 61 | 62 | 66 | 81 | 90 | 87 | 93 | |
| Class | B | B | B | B | A | A | A | B | |
| Mean pressure | 520 | 500 | 500 | 470 | 455 | 455 | 455 | 410 | |

* Position at 1600 :—53° 15′N. 3° 40′W.

Sounding 2 — 28 April 1943 Fazakerley release 1511 G.M.T.

| G.M.T. | 1531* | 1601 | 1631 | 1701 | 1731 | 1801 | 1831 | 1901 | 1931 |
|---|---|---|---|---|---|---|---|---|---|
| Direction | 314 | 310 | 307 | 311 | 297 | 305 | 305 | 293 | |
| Speed km/hr | 80 | 70 | 88 | 83 | 83 | 80 | 80 | 72 | |
| Class | A | B | B | B | B | A | A | B | |
| Mean pressure | 515 | 535 | 490 | 450 | 475 | 425 | 420 | 395 | |

* Position at 1531 :—53° 15′N. 0° 22′W.

Sounding 3 — 6 September 1943 Penzance release 1649 (approx.) G.M.T.

| G.M.T. | 1800* | 1830 | 1900 | 1930 | 2000 |
|---|---|---|---|---|---|
| Direction | 266 | 260 | 259 | 251 | |
| Speed km/hr | 68 | 87 | 98 | 85 | |
| Class | B | A | A | B | |
| Mean pressure | 455 | 420 | 405 | 410 | |

* Position at 1800 :—50° 30′N. 0° 14′W.

Sounding 4 — 24 December 1943 Fazakerley release 1500 G.M.T.

| G.M.T. | 1515 | 1545 | 1615 | 1645 | 1715 | 1745 | 1815 | 1845 |
|---|---|---|---|---|---|---|---|---|
| Direction | 001 | 006 | 009 | 016 | 008 | 013 | 026 | |
| Speed km/hr | 107 | 92 | 117 | 97 | 112 | 111 | 108 | |
| Class | B | B | C | C | A | A | C | |
| Mean pressure | 465 | 490 | 465 | 455 | 460 | 455 | 470 | |
| Position | 53.1N. | 52.6N. | 52.2N. | 51.7N. | 51.3N. | 50.8N. | 50.3N. | |
| | 2.7W. | 2.8W. | 2.9W. | 2.9W | 3.1W. | 3.2W. | 3.3W. | |

Sounding 5 — 7 January 1944 Fazakerley release 1501 G.M.T.

| G.M.T. | 1540 | 1610 | 1640 | 1710* | 1645 | 1715 | 1745 | 1815 |
|---|---|---|---|---|---|---|---|---|
| Direction | 277 | 282 | 287 | | 290 | 287 | 287 | 290 |
| Speed kt | 40 | 37 | 44 | | 32 | 35 | 37 | 41 |
| Class | C | C | D | | C | B | A | A |
| Mean pressure | 525 | 530 | 554 | | 545 | 535 | 530 | 520 |
| Position | 53.3N. | 53.2N. | 53.1N. | | 53.2N. | 53.1N. | 53.0N. | 52.9N. |
| | 2.1W. | 1.5W. | 1.0W. | | 1.2W. | 0.7W. | 0.2W. | 0.3E. |

| G.M.T. | 1845 | 1915 | 1945 | 2015 |
|---|---|---|---|---|
| Direction | 290 | 290 | 288 | |
| Speed kt | 41 | 44 | 40 | |
| Class | A | A | B | |
| Mean pressure | 520 | 510 | 550 | |
| Position | 52.8N. | 52.7N. | 52.6N. | |
| | 0.8E. | 1.4E. | 1.9E. | |

* Overlapping readings from two stations

Sounding 6 18 February 1944 Downham Market release 1500 G.M.T.

| G.M.T. | 1530 | 1600 | 1630 | 1700 | 1730 | 1800 | 1830 |
|---|---|---|---|---|---|---|---|
| Direction | 022 | 021 | 010 | 012 | 017 | 023 | |
| Speed kt | 52 | 63 | 71 | 79 | 60 | 55 | |
| Class | A | A | C | B | C | C | |
| Mean pressure | 500 | 510 | 485 | 470 | 465 | 450 | |
| Position | 53.3N. | 51.8N. | 51.4N. | 50.8N. | 50.2N. | 49.8N. | |
| | 0.1E. | 0.2W. | 0.4W. | 0.6W. | 0.8W. | 1.0W. | |

Sounding 7 27 March 1944 Fazakerley release 1500 G.M.T.

| G.M.T. | 1530 | 1600 | 1630 | 1700 | 1730 | 1800 | 1830 | 1900 |
|---|---|---|---|---|---|---|---|---|
| Direction | 298 | 301 | 305 | 305 | 310 | 315 | 315 | |
| Speed kt | 57 | 50 | 67 | 66 | 60 | 60 | 48 | |
| Class | B | D | B | A | A | B | C | |
| Mean pressure | 380 | 390 | 365 | 370 | 365 | 355 | 370 | |
| Position | 53.2N. | 52.95N. | 52.8N. | 52.5N. | 52.1N. | 51.8N. | 51.4N. | |
| | 2.5W. | 1.5W. | 1.1W. | 0.4W. | 0.3E. | 0.8E. | 1.5E. | |

Class A corresponds to a vector error less than 3 kt
Class B corresponds to a vector error of 3 — 5 kt
Class C corresponds to a vector error of 5 — 10 kt
Class D corresponds to a vector error greater than 10 kt

## APPENDIX II

### Calculation of fluctuation due to eddies

Let the values of the " observed " mean winds be $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$, . . . . . $\mathbf{V}n$ for consecutive half hour intervals, and let the smoothed values be $\mathbf{V}$, $\mathbf{V} + \mathbf{A}$, $\mathbf{V} + 2\mathbf{A}$ . . . . $\mathbf{V} + n\mathbf{A}$ where $\mathbf{A} = d\mathbf{V}/dt$.

Then
$$\mathbf{V}_1 = \mathbf{V} + \boldsymbol{\epsilon}_1' + \boldsymbol{\epsilon}_1''$$
$$\mathbf{V}_2 = \mathbf{V} + \mathbf{A} + \boldsymbol{\epsilon}_2' + \boldsymbol{\epsilon}_2''$$
$$\mathbf{V}_3 = \mathbf{V} + 2\mathbf{A} + \boldsymbol{\epsilon}_3' + \boldsymbol{\epsilon}_3''$$

.....

where $\boldsymbol{\epsilon}_1'$, $\boldsymbol{\epsilon}_2'$, $\boldsymbol{\epsilon}_3'$ . . . are the vector errors involved in the determination of the mean winds, and $\boldsymbol{\epsilon}_1''$, $\boldsymbol{\epsilon}_2''$, $\boldsymbol{\epsilon}_3''$ . . . . are the departures arising from fluctuations due to eddies, *i.e.*, departures arising from the variation of the acceleration from its assumed constant value $\mathbf{A}$.

The departure of $\mathbf{V}_1$ from $\mathbf{V}$ is given by

$$\boldsymbol{\delta}_1 = \boldsymbol{\epsilon}_1' + \boldsymbol{\epsilon}_1''$$

For a large number of observations of $\mathbf{V}_1$, we have

$$\Sigma\, \boldsymbol{\delta}_1^2 = \Sigma\, \boldsymbol{\epsilon}_1'^2 + \Sigma\, \boldsymbol{\epsilon}_1''^2$$

since the $\boldsymbol{\epsilon}''$s and $\boldsymbol{\epsilon}'''$s are independent

$$\therefore\ \mathbf{v}_1^2 = \mathbf{v}_1'^2 + \mathbf{v}_1''^2$$

where the $\mathbf{v}$'s refer to the mean square departures.

Similar expressions can be obtained for the departure of $\mathbf{V}_2$ from $\mathbf{V} + \mathbf{A}$, of $\mathbf{V}_3$ from $\mathbf{V} + 2\mathbf{A}$ and so on. Summing these expressions we have

$$\mathbf{v}^2 = \frac{\Sigma\, \mathbf{v}_1'^2}{n} + \frac{\Sigma\, \mathbf{v}_1''^2}{n} \qquad \text{(A)}$$

where $\mathbf{v}$ is the mean square departure of the observed winds from the smoothed winds for all values of the observed winds throughout the sounding.

An approximation to $\mathbf{v}^2$ is given by $\dfrac{\boldsymbol{\delta}_1^2 + \boldsymbol{\delta}_2^2 + \ldots + \boldsymbol{\delta}n^2}{n}$

$\Sigma \dfrac{\mathbf{v}_1'^2}{n}$ is the mean square departure due to errors in the determination of the mean wind and its values can be calculated from the degree of accuracy reported from the closeness of fit of the position determined by the three D/F stations.

$\Sigma \dfrac{\mathbf{v}_1''^2}{n}$ is the mean square departure due to eddies and can be calulated from the relationship (A)

## Discussion

Mr. Gold : I should like to ask Mr. Durst if he can suggest why the direction obtained from the constant-height balloons is quite different in one case from that obtained by the other method.

Asst. Professor P. A. Sheppard : Having aided a little in realising the idea of constant-height balloon flights for meteorological purposes, I am very glad that the Society has now the opportunity of seeing what advantage Mr. Durst has taken from them.

Fellows may care to know a little more of how the flights were made. At that time the British radio-sound stations observed the wind by radio direction-finding equipment trained on a transmitter carried by the balloon. A constant-height balloon having been released from the most windward station in the British Isles, it was followed by that station until it passed out of range of the station receivers. Meanwhile it would probably have been picked up by the next station downwind in the network and having passed through this station's " field " would be taken over by a third station. That, with the necessarily rather small size of the British network, was usually the limit of the relay. Speaking from memory I believe the maximum path of tracked flight was about 300 mi.

Sitting here tonight I have thought once again how near and yet so far we are from obtaining the trajectories of parcels of air with this technique. The balloon can take no notice of the vertical motion of the air (relative to the pressure level of flight) and while, as Mr. Durst has shown, it can provide valuable meteorological data, it cannot add the great thing missing from our synoptic charts.

Dr. Sutcliffe : I congratulate Mr. Durst on his showing us the value of his constant-level balloon technique. He has established a good case and I hope it will be possible to do more work on these lines. I was rather intrigued by the balloon which passed through a front and should like to know the details. Fronts are thought of as air mass boundaries and one would not expect the air to pass through the front. I do not say this is impossible but it would be interesting to know the circumstances. Perhaps vertical motion supplies the explanation.

Dr. G. D. Robinson : It appears from the conclusions of the paper that the authors differentiate between geostrophic departures associated with minor disturbances of the circulation on a scale of 2 to 4 hours, say 150 to 300 mi, and these associated with eddies on a scale of $\frac{1}{2}$ hr — *i.e.*, about 50 mi. I would like to know if Mr. Durst considers that there is a worthwhile physical distinction between these two scales of pertubation of a larger circulation.

Mr. A. G. Matthewman : The paper seems to imply that upper-air contour charts should be drawn without reference to actual reported winds, which is contrary to present practice. If proven, this implication would be of practical importance, but it must be borne in mind firstly, that the present paper deals with only a very few cases, and secondly that it is probably unfair to compare the accuracy of upper-air research charts with those drawn under routine forecasting conditions in haste.

Mr. J. M. Craddock : After seeing Mr. Durst's diagram, I was surprised to hear that the correlations between the various estimates of the ageostrophic wind

were so low. The estimates of the ageostrophic wind direction seemed to agree very well, and the low correlations must be due to discrepancies between the estimates of wind speed. Has Mr. Durst considered the estimates of ageostrophic wind speed and wind direction separately ?

Mr. R. H. Eldridge : Mention is made of the lack of sensitivity of the mechanism for keeping the balloon at constant pressure height. Could more information be given about the means used, please ?

Mr. C. S. Durst : The passage of the balloon in Ascent No. 3 through the front could be effected by the air in which the balloon was travelling, changing height and leaving the balloon behind (at constant pressure). Thus there was no inconsistency in the balloon passing through a front. That being so, it is clear that no value of the geostrophic departure can be calculated under those particular circumstances.

The control of the balloon's height was by a valve which came into action when expansion brought the balloon to a certain dimension.

The correlation coefficients which had been calculated between geostrophic departures were vector correlations and so took into account both direction and speed.

In my view it is a mistake to draw contour charts which take into account the observed winds. What are really needed are two sets of charts, a set of streamline charts which represent the flow of wind (with isopleths of speed) and a set of contour charts which represent true contours.

The eddies which give rise to short-period fluctuations of wind can be distinguished from the geostrophic departures deduced for periods of two or three hours — the latter are features which are likely to affect the forecasting charts.

551.513.1

# Equatorial circulations

By N. P. SELLICK, M.C., B.Sc., B.Sc. (Eng.)

(Manuscript received 8 November 1948, revised September 1949)

SUMMARY

Grimes (1937) made a first essay to treat the passage of air across the equator dynamically. His work has been extended by Crossley (1948). In the present paper a flow pattern is presented which proves not intractable in spherical polar co-ordinates in the equatorial belt. Frictionless horizontal flow is considered and it is shown that clockwise and counter-clockwise circulations can exist in equilibrium on the equator. The centre of the pressure field is displaced into the hemisphere for which the circulation is cyclonic and both circulations thus appear as cyclones. It is found that trajectories and contours pass freely across the equator but that the relation between winds and contours is variable.

## 1. The equations of motion

The equations to frictionless horizontal motion on a rotating sphere may be written

$$r^2 \cos^2 \phi \, . \, \ddot{\lambda} - 2 r^2 (\dot{\lambda} + \omega) \sin \phi \cos \phi \, \dot{\phi} = - \frac{\partial \psi}{\partial \lambda}$$

$$r^2 \ddot{\phi} + r^2 (\dot{\lambda} + 2\omega) \sin \phi \cos \phi \, \dot{\lambda} = - \frac{\partial \psi}{\partial \phi} \quad . \quad . \quad . \quad (1)$$

in which motion is referred to an isobaric surface. $\lambda$, $\phi$ and $r$ are the three co-ordinates and the customary notation is used, $\psi$ is the dynamic elevation of a surface of constant pressure.

When there is no convergence, absolute vorticity is conserved and the following equation must be satisfied

$$\frac{d}{dt} \left\{ \frac{1}{\cos \phi} \frac{\partial \dot{\phi}}{\partial \lambda} - \cos \phi \frac{\partial \dot{\lambda}}{\partial \phi} + 2 \sin \phi (\dot{\lambda} + \omega) \right\} = O \quad . \quad . \quad (2)$$

It may be remarked that certain conditions must be satisfied in regard to motion about the other two axes of vorticity. These conditions can be satisfied by trifling horizontal temperature gradients and are not dealt with here.

The following non-divergent flow pattern is adopted

$$\dot{\lambda} = a + \frac{a}{\cos \phi} \sin b (\lambda + \beta t) \cos b \phi$$

$$\dot{\phi} = - \frac{a}{\cos \phi} \cos b (\lambda + \beta t) \sin b \phi \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which $a$ provides for a general velocity to the east or west, $\beta$ for translation of the pattern and $a$ for a velocity of circulation. The pattern is one of alternating clockwise and counter-clockwise rotation in cells bounded by lines of latitude and longitude and $b$ determines the size of the cells.

Eqs. (3) may be substituted into the vorticity Eq. (2) and the absolute vorticity $\xi$ is

$$\xi = \left(\frac{1}{\cos^2 \phi} + 1\right) ab \sin b(\lambda + \beta t) \sin b\phi + a \tan \phi \sin b(\lambda + \beta t) \times \cos b\phi + 2 \sin \phi (a + \omega) \quad . \quad . \quad . \quad (4)$$

Differentiation of this expression with respect to time shews that Eq. 2 cannot be satisfied exactly and it is necessary to examine the order of the terms.

The following values are adopted

$$\omega = 7{\cdot}29 \times 10^{-5}$$
$$a = 7{\cdot}84 \times 10^{-7}$$
$$\beta = 0 \; ; \; 1{\cdot}82 \times 10^{-7} \; ; \; 7{\cdot}84 \times 10^{-7}$$
$$b = 6{\cdot}38 \times \pi = 20$$

The cells are then 9° square, roughly 1,000 km, and the maximum velocity of translation of the pattern and of circulation is 5 m/sec. The pattern will not be extended beyond 9° from the equator and the order of magnitude of the three terms is

$$3{\cdot}14 \times 10^{-5} \sin b\lambda \sin b\phi$$
$$1{\cdot}24 \times 10^{-7} \sin b\lambda \cos b\phi$$
$$2{\cdot}28 \times 10^{-5} + 0{\cdot}316\, a$$

It is evident that the middle term is very small and could be dropped. Differentiating Eq. (4) with respect to time confirms that terms deriving from the middle term are small and

$$\frac{d\xi}{dt} = a \cos b\lambda \left\{ (a + \beta) \left(\frac{1}{\cos^2 \phi} + 1\right) b^2 \sin b\phi - 2 \sin b\phi (a + \omega) \right\}$$

very nearly.

Note—$\lambda + \beta t$ is written $\lambda$ for brevity.

Equating this expression to zero and taking $\cos \phi = 1$

$$a + \beta = \frac{a + \omega}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

an expression for the velocity of motion of the air through the pattern.

Eqs. (3), entered into Eqs. (1), provide the following results directly

$$-\frac{1}{r^2}\frac{\partial \psi}{\partial \lambda} = (a + \omega) \cos b\lambda \left(\frac{a}{b} \cos \phi \cos b\phi + 2a \sin \phi \sin b\phi\right) + a^2 b \sin b\lambda \cos b\lambda + a^2 \tan \phi \sin b\lambda \cos b\lambda \sin b\phi \cos b\phi$$

$$-\frac{1}{r^2}\frac{\partial \psi}{\partial \phi} = (a + \omega) \sin b\lambda \left(\frac{a}{b \cos \phi} \sin b\phi + 2a \sin \phi \cos b\phi\right) + \frac{a^2 b}{\cos^2 \phi} \sin b\phi \cos b\phi + a(a + 2\omega) \sin \phi \cos \phi + a^2 \tan \phi \left(\frac{1}{\cos^2 \phi} \cos^2 b\lambda \sin^2 b\phi + \sin^2 b\lambda \cos^2 b\phi\right)$$

Introducing numbers it is found that the terms in $a^2 \tan \phi$ are of the order of $10^{-2}$ of the remainder of the terms and the equations can be written, approximately

$$-\frac{1}{r^2}\frac{\partial \psi}{\partial \lambda} = (\alpha + \omega) \cos b\lambda \left(\frac{a}{b} \cos b\phi + 2a \sin \phi \sin b\phi\right) + a^2 b \sin b\lambda \cos b\lambda$$

$$-\frac{1}{r^2}\frac{\partial \psi}{\partial \phi} = (\alpha + \omega) \sin b\lambda \left(\frac{a}{b} \sin b\phi + 2a \sin \phi \cos b\phi\right) + a^2 b \sin b\phi \cos b\phi$$

$$+ \frac{a}{2}(\alpha + 2\omega) \sin 2\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\cos \phi$ has been taken as equal to 1.

These expressions are integrable and the equation to the contours is

$$-\frac{1}{r^2}\psi = (\alpha + \omega)\frac{a}{b^2} \sin b\lambda \cos b\phi + \frac{a^2}{2} \sin {}^2b\lambda + \frac{a^2}{2} \sin {}^2b\phi$$

$$+ (\alpha + \omega)\frac{2a}{b} \sin \phi \sin b\lambda \sin b\phi + \frac{a}{2}(\alpha + 2\omega) \sin {}^2\phi - \frac{1}{r^2}\psi_0 \quad . \quad (7)$$

Finally, from Eqs. (3)

$$\alpha \sin \phi + \frac{a}{b} \sin b\lambda \sin b\phi = const. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

## 2. Specimen patterns

Eq. (7) can be solved as a quadratic in $\sin b\lambda$ and, utilizing the constants given earlier, contours at convenient intervals may be found. Eq. (8) enables streamlines to be found.

Calculations have been made for the following cases :

(a) The pattern is bisected by the equator and extends to 4·5° to north and south. The wind and contour fields have been computed for

(i) $\beta = 0{\cdot}0$

(ii) $\beta = 1{\cdot}82 \times 10^{-7}$

(iii) $\beta = 7{\cdot}84 \times 10^{-7}$

(b) The pattern is based on the equator and extends to 9° N. An extension to 9° S. would be a mirror image of the pattern. In this case

$$\beta = 1{\cdot}82 \times 10^{-7}$$

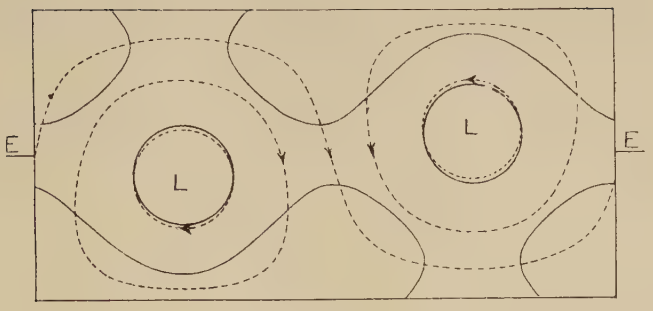
Figure 1 (*a*). In this pattern $\beta = 0$ and the contours and streamlines are stationary ; the latter are therefore trajectories. The wind system in the left hand cell is compounded of a clockwise circulation and a small velocity to the east. The eastward velocity is 1·16 m/sec. This motion of the air through the pattern is required to maintain the vorticity constant along a trajectory and was first remarked by Rossby (1939). The addition of a translatory velocity has moved the centre of rotation to the south of the equator and the streamlines appear as a wave crest below which is a closed circulation. The contours show a centre of low pressure to the south of the equator with approximately circular contours extending to about 3° north of the equator. The winds and pressure field for a radius of 2 or 3° from the centre of low pressure have the appearance of a southern hemisphere depression, the agreement between the inner contour and streamline is very close. Beyond this radius there is a weak trough along the equator. There is a col centre at about 3° north. If the pattern were extended north the next cell would have a counter-clockwise circulation and a low pressure centre. The fringe of this low pressure appears on the northern side of the col. To the east and west of the col there are patches of high pressure which are not associated with the customary wind system at all. It is interesting to note the simple solution achieved in regard to clockwise and counter-clockwise spin, — both are cyclonic.

Figure 1 (*b*). In this pattern the air has no translatory velocity and the streamlines are symmetrical clockwise and counter-clockwise cells. The contours are displaced to north and south to the hemisphere in which the circulation is cyclonic. Lobes of high pressure again appear over the area containing the circulation col. The broad stream of air crossing the equator is noteworthy. The pattern must translate to the west with a velocity of 1·16 m/sec.

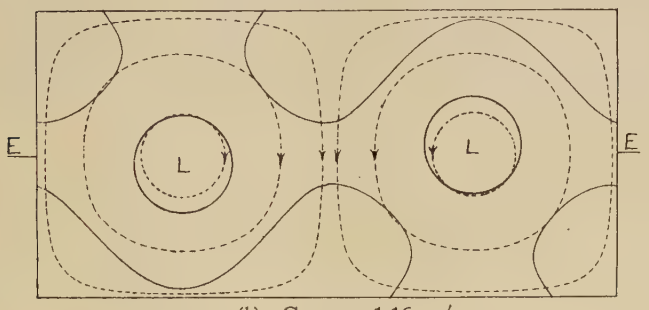
Figure 1 (*c*). In this pattern the streamlines are distinctly wave form but the cyclonic character is preserved to a considerable extent in both hemispheres. The inclination of the streamlines to the contours is very marked.

Figure 1 (*d*). In this pattern the equator is on the lower edge and anticyclonic circulation appears. The solution is rather interesting — the contours bear no relation to the streamlines. The cyclonic circulation on the other hand resembles the classic pattern very closely.

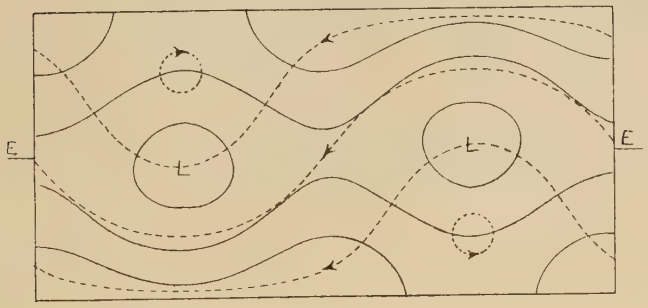
The cellular wind pattern adopted above is convenient in that it provides a solution to the problem of change of vorticity with latitude. It has the disadvantage that no means have been found to develop the pattern from still air by divergence. On the other hand it closely resembles solid rotation near the centre and a col at the intersections. It appears unlikely that its artificiality detracts seriously from the general conclusions which may be drawn from the diagrams.



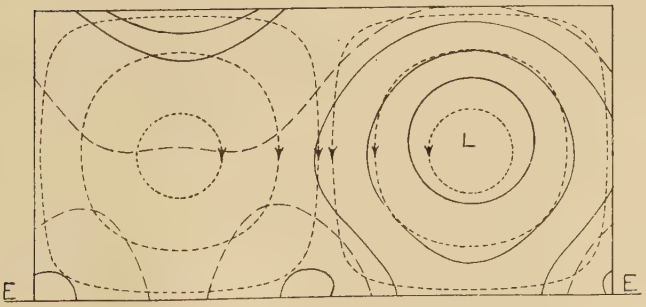

(*a*) C = 0




(*b*) C = — 1·16 m/s




(*c*) C = — 5·0 m/s




(*d*) C ≐ — 1·16 m/s

Figure 1. For explanations, see text.
C indicates velocity of the translation of the pattern.

## Conclusions

A number of useful conclusions can be drawn :

1. No physical difficulty exists to broad streams of air crossing the equator at any angle. Contours may also cross the equator at any angle.

2. Cyclonic contours are dominant. They may exist across the equator and are likely to be associated with cyclonic circulations rotating in accord with the hemisphere in which the pressure centre lies and following the contours with reasonable fidelity except when the system is translating rapidly. (It remains to be explained why, in the latitudes of Southern Rhodesia, anticyclonic circulations dominate the cols.)

3. Anticyclonic circulation can exist near the equator but the contours associated with the circulation bear no resemblance to the classical model. It is not clear how such circulations can arise.

4. Patches of high pressure appear to be associated with the centres of cols rather than with anticyclonic circulations and pressure cols do not coincide with cols in the circulation.

5. Little relation can be expected between contours and wind direction except near the centres of low pressure areas and not there if the translation is appreciable.

## References

Grimes, A. 1937 *Mem. Malayan Met. Service*, Singapore, No. 2.
Haurwitz, B. 1940 *J. Mar. Res.*, Newhaven, Conn., **3**. p. 254.
Rossby, C. - G. 1939 *Ibid.*, **2**, p. 38.

551.593.5 : 535.34

# The effect on observations of the sky/sea brightness ratio produced by the use of colour filters and Polaroid screens

By D. R. BARBER
(*Norman Lockyer Observatory, University College of the South-West*)

(Manuscript received 16 July 1949)

SUMMARY

Visual and photographic observations of the illumination of the sky-sea horizon, made in light of different wavelengths, indicate an enhanced brightness contrast when the shorter wavelengths are excluded. The effect is most pronounced in the presence of haze, or light mist. When use is made of an analysing filter to modify the relative amounts of polarised light received from sky and sea, the degree of contrast obtainable depends on a number of independent factors, *e.g.*, amount of cloud cover, and state of sea surface that may, or may not, combine to give better visibility of the sky-sea boundary. Anomalous polarisation effects are frequently observed during early twilight along the anti-solar path. This work was carried out while with the Kodak Research Laboratories. The paper is issued as Communication No. 1263H from the Kodak Research Laboratories.

## 1. INTRODUCTION

It was desired to ascertain whether or not the use of colour filters, or polarising screens, would be efficacious in improving the visibility of craft at sea silhouetted against the sky/sea horizon. Some experiments were therefore carried out during the early summer of 1943 as part of a detailed investigation of the brightness contrast at the horizon boundary under a variety of meteorological conditions and at different illumination levels. The work was carried out by the Kodak Research Laboratories under contract with the Admiralty.

## 2. EQUIPMENT

The visual observations which form a part of the experimental data to be discussed later were all made with a G.E.C. portable telephotometer of Luckiesh-Taylor type. They were supplemented by photographic observations made with the aid of a Zeiss 7 cm, F3·5 anastigmat lens, and a wedge calibrator device fitted into the body of the camera. The colour filters used for the tests were Wratten Nos. 4 (light yellow) ; 21 (deep yellow) ; 29 (deep red) ; a Russian yellow glass filter supplied by the Admiralty Research Laboratory, and a Polaroid (commercial quality) glass screen. In addition, a set of Ilford Trichromatic filters was used to obtain a photographic record of the sky/sea boundary in blue, green and red light.

## 3. OBSERVATIONS

The results discussed in the next section are based on observations made in one direction only, azimuth 180°, over a range of atmospheric transmission values from 95 to 65 per cent 1,000 yd. The times of observing were chosen so as to include the effects of varying degrees of cloud cover and sea roughness. Normally the routine procedure was to make two independent observations of the sky/sea

brightness ratio with each colour filter and, in addition, two observations without filter as rapidly as possible. The non-filter observations were included so as to serve for a basis of comparison.

When photographing in polarised light a pair of observations was made, the first of the pair with the polarising screen so orientated that the maximum amount of sky-light was transmitted, and the second with the screen rotated through 90 degrees from its former position.

## 4. Discussion of results

An analysis of the observations made visually and photographically with the set of colour filters revealed a marked difference in their efficiency as regards contrast enhancement. Whereas filters W.4 and W.21 offered no improvement over observations made without a filter and, in certain types of directional lighting occasionally produced a lowered contrast, filters W.29 and A.R.L. yellow were found to effect a significant increase of contrast, more particularly when using the red filter.

The relative efficiencies as measured appear in the following Table I.

TABLE I. Percentage gain or loss in contrast with various filters

| Colour Filter | Percentage Transmission per 1000 yd | | | | | | |
|---|---|---|---|---|---|---|---|
| | 95 | 91 | 88 | 85 | 79 | 67 | Average |
| W.4 | —17 | —11 | + 9 | +13 | — 3 | — 3 | — 2 |
| W.21 | —12 | — 1 | + 2 | +17 | — 7 | — 2 | + ½ |
| A.R.L. | — | — 1 | +10 | +11 | + 7 | + 1 | +5½ |
| W.29 | — 4 | +26 | +23 | +25 | +25 | +17 | +19 |

It is at once apparent that no significant gain in contrast results from the use of filters W.4 and W.21 either photographically or visually, whereas A.R.L. yellow and W.29 filters, show a significant gain, the contrast increase for the deep red filter, *viz.*, 19 per cent, being particularly useful in very misty conditions. No attempt was made to extend the wavelength band by the use of an infra-red plate and filter since, of necessity, the practical application of any enhanced contrast ratio was confined to visual light.

Supplementary photographic records on panchromatic film, made through a set of standard tricolour filters, showed a general agreement with the main series of observations, and confirmed the increase in contrast obtained by confining the colour of the exposing light to the longer wavelengths.

The results of the parallel set of observations with the aid of a polarising filter indicate that the contrast obtainable is influenced by a number of independent factors that may, or may not, combine to yield a practical enhancement of the brightness ratio at the sky/sea boundary. Under conditions of clear blue sky, and calm sea, the maximum variation in contrast is found to occur at azimuths approximately 90° to sun, or twilight path, *i.e.*, those regions of the sky for which the polarised light will have a maximum value. Whilst, in general, for cloudless

skies, or skies with few broken clouds, a gain in filter efficiency results when the analysing filter is set to transmit a greater amount of polarised sky light than that which is reflected from the surface of the sea, the resultant effects are often quite unpredictable, and appear to be affected greatly by local conditions of cloud reflection and state of the sea surface. Anomalous effects are also frequently observed during early twilight, within the twilight path where the earth's shadow forms a background to the sky/sea boundary. At such times a narrow, dark line, which appears typical of the twilight scene, separates a darker sky region within the shadow area from the much lighter sea surface (Figs. 1, e and f)

One particularly interesting set of observations made during early twilight provides useful data on the effect of the advancing earth's shadow. When readings of sky brightness were made close to the sea horizon, *i.e.*, totally within the shadow area, the observed values of the brightness ratio were 1·22 when the filter was set to transmit the polarised sky-light more freely, and 0·78 when set to transmit light polarised by reflection from the sea surface. When the sky brightness values were derived from an area immediately above the earth's shadow, at an altitude of about 10°, the values of the brightness ratio were found to be much higher, *viz.*, 2·64 and 2·00 respectively. It was further observed that the light which originated within the shadow area was more strongly polarised than that scattered from the twilight sky above, the angle of polarisation being rotated through 90°. The respective polarisations at an azimuth of 180° were 8 and $2\frac{1}{2}$ per cent. In general, for overcast skies, a reverse effect is found, a significant increase in the value of the contrast ratio occurring when the analysing filter is oriented so as to transmit a smaller proportion of the polarised sky light. Increases of from 10 to 50 per cent are common and in one instance only was the observed increase so small as to be insignificant. The results obtained for a typical series of records made with the polarising filter at different transmission values are summarised in Table II.

TABLE II. PERCENTAGE GAIN OR LOSS IN CONTRAST WITH POLARISING FILTERS

| Orientation of Polaroid filter | Percentage Transmission per 1000 yd | | | | | | |
|---|---|---|---|---|---|---|---|
| | 95 | 95 | 91 | 88 | 85 | 79 | 67 |
| Sky ‖ | —17 | +14 | +10 | —18 | + 3 | ± 0 | — 6 |
| Sky = | +17 | + 4 | —10 | + 4 | +46 | +23 | +13 |

It is of practical interest to discuss in greater detail the data for two sets of observations representative of clear and overcast sky conditions which demonstrate the " reversal effect " already alluded to. The analysis will be found in Table III. Here brightness values are expressed in candles/ft$^2$ and the values of the percentage polarisation have been reduced by means of the equation,* $p = (1 - K)/(1 + K)$

* The value of $p$ so formed is only an approximate one, since the Polaroid filter is not 100 per cent efficient when analysing plane polarised light.

where $K$ is the depolarisation factor, *i.e.*, the ratio of the transmitted intensities corresponding to the ═══ and ‖ positions of the analysing filter. It will be noted that, under a clear sky, the light reflected from the sea is less strongly polarised than is the incident sky light, whilst, for overcast skies, the reverse is the case. In this latter instance an approximately two-fold increment of the plane-polarised component of the light reflected from the water surface was observed whereas, under a clear blue sky, the ratio of sky/sea polarisation was 1·4 only. Further, a rotation of the analysing filter through 90° produces a larger decrease in sea brightness, as seen through the filter, than occurs for the polarised light which is scattered from the sky. In the example just cited the ratios were found to be 1·7 and 1·3 respectively. An explanation is thus offered for the necessity to rotate the filter through 90° when observing in cloudy weather in order to obtain an enhanced brightness ratio at the sea horizon. The effects obtained under the meteorological conditions discussed above are illustrated in the photographic reproductions of Figs. 1a to 1d.

TABLE III.

| | Cloudless Sky | | | Overcast Sky | | |
|---|---|---|---|---|---|---|
| | No filter | Polaroid ‖ | Polaroid ═══ | No filter | Polaroid ‖ | Polaroid ═══ |
| Sky brightness c/ft² | 598 | 565 | 315 | 44 | 42·7 | 34 |
| Percentage polarisation | — | 28·5 | | | 11·5 | |
| Sea brightness c/ft² | 342 | 300 | 202 | 35 | 34 | 20·6 |
| Percentage polarisation | — | 19·5 | | | 24·5 | |
| Sky/sea brightness ratio | 1·75 | 1·89 | 1·56 | 1·25 | 1·26 | 1·65 |
| Percentage gain or loss | | + 8 | —11 | | + 1 | +32 |

## Conclusions

Summarising the results of the tests with the various filters used in conjunction with visual and photographic measurements of the brightness ratio of the sky/sea boundary it has been found that, of the four colour filters tested, only two, a deep red Wratten 29 and A.R.L. Russian Yellow, produce any significant gain in contrast; use of the first-named filter produces an increase of the order of 20 per cent.

The results obtained with the polarising filter are of somewhat doubtful value for clear sky conditions ; but, for overcast skies, especially when associated with appreciable amounts of haze or mist, the contrast ratio is raised by amounts of from 10 to 50 per cent of the unscreened value. A well-marked and characteristic " reversal effect " in polarised light is observed during early twilight which results from the encroachment of the earth's shadow on the area of sky immediately above the sea horizon.

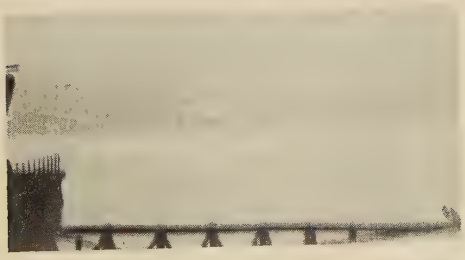
ORIENTATION OF POLAROID FILTER

SKY

SKY

(*a*) CLEAR (*b*)

(*c*) OVERCAST (*d*)



(*e*) TWILIGHT (*f*)

Figure 1. Photographs showing sky/sea contrast using filters.

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# A note on the relation between ageostrophic winds and isallobaric gradients

By M. K. MILES, M.Sc., A.R.C.S., D.I.C.

(Manuscript received 12 September 1949)

Summary

Using the observational material presented in an earlier paper (Miles 1949) it is found that the following relation fits the observations ahead of most of the warm fronts :

$$|A| = 1{\cdot}45\ I_M - 1$$

where $|A|$ is the magnitude of the ageostrophic wind at 900 mb and $I_M$ the measured isallobaric wind as defined by Brunt and Douglas (1928), both in mi/hr.

It is shown that when allowance is made for the fact that $I_M$ is obtained from isopleths drawn from three-hour tendencies the result compares closely with other observational relations which have been shown (Miles 1949) to be in agreement with theory.

This and some further measurements from synoptic charts lead to the conclusion that in the majority of cases the measured isallobaric wind can give a useful first approximation to the ageostrophic wind at 900 mb.

## 1. Introduction

It seemed probable from earlier work (Miles 1949) that, a few occasions apart, there might be a fairly close relation between the ageostrophic wind and the isallobaric gradient. Should this be the case it is a matter of considerable practical and theoretical interest to find out as nearly as possible what the relation is. Isallobars had been drawn on about half of the charts examined previously and it was decided to draw them on the remainder and see what could be got from the complete data.

## 2. Observations and results

Isallobars were drawn at 1 mb/3hr intervals and the isallobaric wind as defined by Brunt and Douglas (1928) was obtained by putting a geostrophic scale on these isopleths and multiplying the equivalent geostrophic wind by 0·83 : this gives the result in mi/hr if the geostrophic scale is graduated in mi/hr. The isallobaric winds obtained in this way must be considered to be subject to errors up to about $\pm$ 4 mi/hr. The line of best fit for about half only of the cases studied had previously been obtained (Miles 1949). Fifty-three values of this quantity, which we will call $I_M$, were now obtained and plotted against the appropriate ageostrophic wind $|A|$. It is clear that the results for the warm fronts of 3 February 1946 and 11 December 1946 are exceptional and the form of the isobars for the latter system suggests that the field terms are likely to be large. For the former it is difficult to assess them but it appears likely that they are opposite in sign to those of 11 December which agrees with the position of the points on the scatter diagram. When the four points for these two situations are omitted a correlation coefficient of + 0·69 is found to exist between $|A|$ and $I_M$. The scatter diagram for these forty-nine cases is shown in Fig. 1 with the line of best fit :

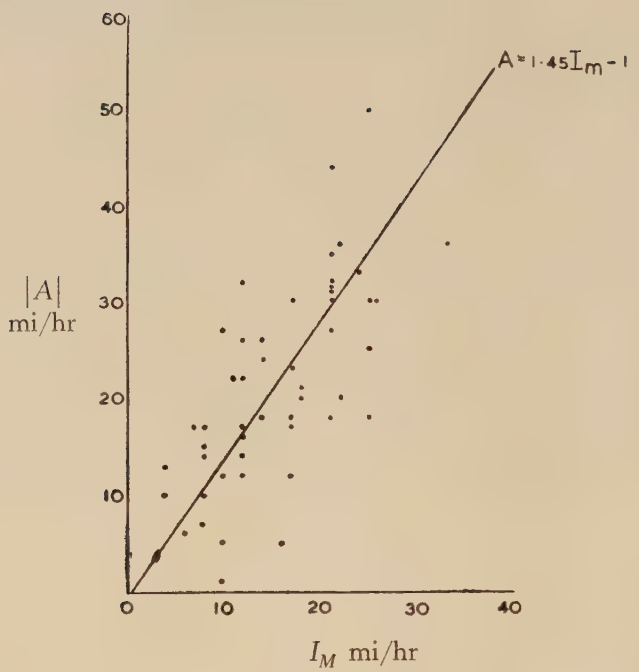


Figure 1. Scatter diagram of ageostrophic wind against isallobaric wind.

$$|A| = 1{\cdot}45\ I_M - 1 \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

drawn in. The points have a standard deviation of 4·5 mi/hr from this line.

## 3. Discussion of results

In the previous paper it was found possible to explain many features of the observed ageostrophic motion by means of a theoretical model in which the pressure fall followed an exponential rate $e^{-kt}$ and an approximate value for $k$ was $\lambda$ the Coriolis parameter. Because the measured isallobaric wind is based on pressure changes over 3 hr, the measured values $I_M$ differ from the true values corresponding to the instantaneous values of the pressure tendency. For the exponential rate of change indicated above and from the definition of $I_M$,

$$I_M = 1{\cdot}35\ \frac{\partial J}{\partial t} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $J$ is the geostrophic wind.

Using this relation Eq. (1) can be written

$$|A| = 1{\cdot}95\ \frac{\partial J}{\partial t} - 1 \qquad , \quad , \quad , \quad , \quad , \quad (3)$$

The line of best fit for observations* of $|A|$ and the six-hour change in the geostrophic wind at a single station has been found to be

$$|A| = 0{\cdot}8\ (J_t - J_{t-6}) + 2 \qquad . \quad . \quad . \quad . \quad (4)$$

With the same assumption about the rate of growth of the pressure gradient this can be expressed as

$$|A| = 1{\cdot}84\ \frac{\partial J}{\partial t} + 2 \qquad . \quad . \quad . \quad . \quad . \quad (5)$$

(In a uniform pressure gradient, $\partial J/\partial s = 0$, with a known growth rate, $J_t - J_{t-6}$ and $I_M$ both measure the rate of change of the gradient). A comparison of Eqs. (3) and (5) suggests that terms involving $\partial J/\partial s$ (the field change) are usually small compared with $\partial J/\partial t$, and with plus and minus values of nearly equal frequency.

This conclusion receives support from the relation between $|A|$ and $dJ/dt$ which can be derived from the experimental result, given in the previous paper, $|A| = 0{\cdot}78J' - 1$ where $J'$ is the magnitude of the vector change in the geostrophic wind along a trajectory over 6 hr. Using the same assumption about the rate of growth, this relation can be expressed as

$$|A| = 1{\cdot}80\ \frac{dJ}{dt} - 1 \qquad . \quad . \quad . \quad . \quad . \quad (6)$$

Expanding $\frac{dJ}{dt}$ as $\frac{\partial J}{\partial t} + V\ \frac{\partial J}{\partial s}$ where $V$ is the wind speed and $\frac{\partial J}{\partial s}$ the variation of $J$ along a trajectory we get

$$|A| = 1{\cdot}80\ \frac{\partial J}{\partial t} + 1{\cdot}80\ V\ \frac{\partial J}{\partial s} - 1 \qquad . \quad . \quad . \quad (7)$$

The relation on which Eq. (7) is based has been shown (Miles 1949) to be in close agreement with the theoretically predicted result and the closeness of the coefficients of $\frac{\partial J}{\partial t}$ in the three cases together with the small values of the constant terms strongly support the view that the term in $\frac{\partial J}{\partial s}$ is normally small compared with that in $\frac{\partial J}{\partial t}$.

The difference in the standard deviation of points from the lines represented by Eqs. (5) and (7), namely 4 mi/hr, gives some indication of the usual magnitude of the term in $V\ \frac{\partial J}{\partial s}$. Taking $V$ to be about 30 mi/hr this makes $\frac{\partial J}{\partial s}$ about 0·07 mi/hr mi$^{-1}$.

$\partial J/\partial s$ has been determined from surface isobaric charts for 26 occasions and the mean without regard to sign is 0·06 mi/hr mi$^{-1}$. Eight were positive, thirteen negative and five zero.

* This has been taken as the bisector of the two regression line

## 4. Conclusions

With the majority of the warm fronts the contribution of the term in $\partial J/\partial s$ to the ageostrophic wind is likely to be less than 5 mi/hr ; with positive and negative values of nearly equal frequency. In cases where the contribution of this term is likely to exceed 5 mi/hr it is often possible to see by inspection that there is marked divergence or convergence of the isobars which is likely to be maintained, as for example when a warm front moves towards a quasi-stationary anticyclone (*cf.* 11 December 1946).

Thus in general a good working approximation to the ageostrophic wind is provided by the relation

$$|A| = 1{\cdot}9 \frac{\partial J}{\partial t}$$

If $\partial J/\partial t$ is approximated by a finite difference in $J$ over, say, six hours the relation becomes

$$|A| = 0{\cdot}83 \, (J_t - J_{t-6})$$

or if the isallobaric wind, determined from three-hour pressure tendencies is used the relation is

$$|A| = 1{\cdot}4 \, I_M$$

## References

Brunt, D. and Douglas, C. K. M. 1928 *Mem. R. Met. Soc.*, London, **3**, No. 22.
Miles, M. K. 1949 *Quart. J. R. Met. Soc.*, London, **75**, p. 399.

551.508.71

# DISCUSSIONS

## (*a*) A portable indicating apparatus for the study of temperature and humidity profiles near the ground.

## (*b*) Some estimates of the amount and diurnal variation of evaporation from a clayland pasture in fair spring weather.

By F. PASQUILL, B.Sc., F.Inst.P.

(Read 16 November 1949. See *Q.J.*, **75**, pp. 239 and 249)

### Discussion

Mr. E. Gold : It is very satisfactory to have the evaporation from the earth's surface related to the actual physical conditions of the atmosphere immediately above it. Mr. Pasquill refers in his paper to "Absolute Humidity" and "Specific Humidity." I should like to suggest that these terms should be replaced by "Vapour Density" (or "Water Vapour Density") and "Moisture Content" respectively. The former terms are obscure and confusing : the latter are self-explanatory and their adoption would leave the well established term "Relative Humidity" and if it were abbreviated to "Humidity" no confusion would arise.

Dr. H. L. Penman : I think so highly of the work that Mr. Pasquill has described that I shall probably copy his experimental arrangement for some work at Rothamsted next summer and shall benefit from his second paper in analyzing results from measurements of temperature, humidity and wind profiles that we have been recording during the past two years.

There are three points, however, on which I should like his comments, or on which further explanation would be welcome.

First, is ventilation of the psychrometer an essential part of the technique ? Its sole justification is to permit use of the same psychrometer constant throughout, a complete justification in routine work, but not so essential in research. Its main disadvantage is the risk of disturbing the conditions being measured, particularly in near-calm conditions, and near the ground where the humidity gradient is changing rapidly with height. In this region ventilation will tend to give an over-estimate of the absolute humidity and hence to over-estimate humidity gradients and the derived estimated evaporation rates. The alternative is to measure wind speed at the same level and use the appropriate psychrometer constant for that wind speed, a method we have been forced to use this summer in measuring gradients inside a growing crop where even slight forced ventilation was undesirable. In the open such fussiness may be completely unnecessary : can Mr. Pasquill say if this is so, or not ?

Second, how confident is Mr. Pasquill that Eq. (6) (p. 251) is valid out-of-doors, bearing in mind (i) that nearly all measurements of evaporation out-of-doors during the past 75 years have led to an equation of the form $E \propto (e_1 - e_2)(a + b\,u)$, where $e_1$ is the value at the surface ; and, (ii), that in the paper under discussion there is no direct measurement of evaporation to set alongside the theoretical estimates ?

A third consideration might be added — not too seriously — that in the curves of Fig. 1 both $u_2$ and $(e_1 - e_2)$ are shown as falling to near-zero values together, and Nature does not usually provide two mechanisms for doing the same job.

Third, arising out of the second, I feel unhappy about Eq. (6) because it is formally in conflict with experience. Can it be that the basic equations are unsatisfactory ? As a non-mathematician I am disturbed by the ease with which " constants " such as $k$ and $z_0$, cease to be constants once a differential equation has been solved, and a very wide range of experimental values of these constants was reported two years ago by Prof. Sheppard. In addition to $k$ and $z_0$ there is another parameter, $\tau_0$, in Eq. (1), always assumed constant, and I find it difficult to give a meaning to a shearing stress at the surface when the wind velocity is assumed (by Eq. (1)) to have fallen to zero at a finite height ($z_0$) above the surface. Is the time not ripe for a critical survey of these and other equations of turbulent transport to sort out the physically reasonable from the mathematically convenient ?

Asst. Professor P. A. Sheppard : I wish first to say that in my opinion the work which Mr. Pasquill has described tonight and other observations made by him with the apparatus he has described are some of the most beautiful ever to have been undertaken in this field. We should, I think, be proud to count him among the ranks of British meteorologists.

It is not for me to answer Dr. Penman's question to Mr. Pasquill but he has drawn me into his queries so I would like to say a word on the basis of the equation for evaporation used by Mr. Pasquill, or rather on the equation for momentum exchange with which it is related. This equation, $u = (v_*/k) \log z/z_0$ where $u$ is the wind speed at height $z$, $v_*$ the square root of the surface drag divided by the air density, and $k$ and $z_0$ are constants, is valid under adiabatic conditions over a particular surface. Now the text-books are liable to present this equation as a consequence of the so-called mixing-length theory of turbulence and in doing so probably convince few of their readers. Its essential validity rests however on observation, not merely in the atmosphere but in the fluid-motion laboratory too, the same value of $k$ emerging in both cases. Thus to use it is not to make use of a rather unsatisfying theory but to proceed from a sound but empirical basis. The theory remains to be developed. The same is true of Mr. Pasquill's equation for evaporation and it is he who has established the validity empirically in a paper published elsewhere.

I have regretted the introduction of a variable $k$, to take account of not too large departures from adiabatic conditions, almost ever since suggesting the concept. Again its validity is entirely empirical and an adequate theory will ultimately present the equation of momentum transfer otherwise (Mr. E. L. Deacon has already improved on the empiricism, in a recent paper in this *Journal*).

I cannot too strongly emphasise that no wind or humidity profile equation which is valid in the turbulent flow régime proper, at some distances from the surface, can be expected to have validity very near the surface (within and just above the roughness elements). The characteristic flow pattern just does not exist in the latter region. It is therefore a misleading mathematical device to

adjust a profile equation to make $u$ vanish at a given low height, *e.g.*, $z = o$ or $z_0$. The profile equation given above has an established validity only for $z >> z_0$.

I am not yet ready to accept Dr. Penman's equation, $E = a + b\,u$, to represent evaporation from *natural* boundary surfaces with which we are here concerned, since I believe it to be based on evaporation tank and similar technique, possessing little relevance to the meteorological problem.

In conclusion I would point out a practical advantage in Mr. Pasquill's procedure for overcoming the effects of stable and unstable temperature gradients — to take observations quite near the surface. In doing so there is much less chance of obtaining unrepresentative values of wind speed and humidity due to different kinds of surface at a distance from the site.

Mr. C. F. Lapworth : Since 1942, when Thornthwaite and Holzman first published the formula referred to in the paper, I have been waiting for someone to apply it to the measurement of evaporation from land areas in this country.

As a consulting engineer interested in water supplies, I welcome this approach to the problem. Information is particularly required on the relative amounts of evaporation which can be expected from various types of land surface such as grass, crops and fallow land.

Before going further, however, I would like to see the results obtained from the application of the theoretical formula checked by some alternative method of measurement, such as can be obtained from say an evaporation tank or a percolation gauge.

The method outlined in the paper involves at least six different measurements every hour of the day, a total of 144 measurements per day. The labour involved both in measurement and computation is considerable, and I would like to know whether the author can suggest any short cut which would reduce this.

Dr. J. Glasspoole : It will be seen that a strikingly large diurnal variation is shown for evaporation in Fig. 1 for May 1947 ; there were some experiments made at Valentia (in SW. Ireland) in 1930 that gave a much more uniform variation. In this case the evaporation was measured in an evaporation tank, the mean values (fairly typical of the monthly values) being 1·41 mm for the day (9-18 hr *i.e.*, 9 hr) and 1·25 mm for the night (18-9 hr *i.e.*, 15 hours). While the diurnal variation of wind and vapour pressure would be more uniform at Valentia, it is not easy to see how these results and those of the present paper can be brought into accord. It may be that the apparent expansion of water in the tank has an effect, since with an iron tank the change in water level with a 5°C change of temperature (typical of a warm sunny day) would be of the order of 0·4 mm, *i.e.*, the water level would be lower at the end of the night period and this would appear as evaporation. Another possibility is that the time-lag in the temperature changes of the water of the tank results in more uniform evaporation during the day and night periods. In applying the present paper to the evaporation from a water surface this last factor needs consideration and it would be helpful to have observations by the method of this paper from a water surface. The point is of importance because observations at Kew, I understand, show a smaller diurnal variation than given by Fig. 1.

The existing records of evaporation from evaporation tanks, percolation gauges and those obtained by considering rainfall minus run-off data, are reasonably consistent, when the different factors influencing the results are taken into account. From such records it may well have been anticipated that the evaporation for days in May 1947, as given in Table 1, would be of the order of 1-4 mm. The present uncertainty in estimating the evaporation for a reservoir or area with different types of cover, might be as much as 10-20 per cent and the aim is to reduce this uncertainty by further experiments and by work such as that which Mr. Pasquill has undertaken. It is very important from the water engineering aspect to have reliable estimates for the monthly and annual losses by evaporation in different parts of the country and especially in abnormally dry years.

Dr. G. D. Robinson : Since the question of the value of evaporation tank readings has been raised it might be useful for me to mention the very tentative conclusions drawn from some observations made at Kew Observatory by Mr. N. E. Rider and myself. In these the readings of a recording evaporation tank (the recorder was designed by Mr. L. H. G. Dines) were compared with the rate of evaporation given by a computation in some respects like that of Mr. Pasquill, using wind, temperature and humidity observations made on an apparatus very similar to his. Both tank and profile apparatus were in similar surroundings — a level surface of closely cut grass. The readings were not numerous, and were made at various dates between June and November in 1948 and 1949 always in clear sky conditions. They seem to show that so long as the grass was green and healthy the total evaporation in daylight was about the same from the grass (computed) and the tank (observed). The rate of evaporation from the grass (computed) was higher than that from the tank before about 1400 hr G.M.T., and lower later in the day. At night the computed evaporation sank to almost zero or at times was negative ; the tank frequently continued to evaporate at a very fast rate. When the grass became brown and dry after a prolonged hot spell the computed evaporation in daylight fell considerably below the evaporation observed from the tank. Although no integrated figures of computed evaporation are available it is clear that evaporation tank readings in a dry, hot summer are considerably higher than the natural evaporation from open grassland, but are not entirely useless if night and day totals are taken separately.

Dr. Sutcliffe : As regards the relationship between evaporation from tanks and from natural surfaces it is obvious that in any physical approach to the problem important parameters must be the vapour pressure and the temperature at the surface. The differences between the two surfaces must be considerable.

It would appear that Mr. Pasquill's observations and theory afford a method of determining the deposit of dew at night. This is a very important quantity in the theory of radiation fog, about which we know very little. I should therefore like to ask Mr. Pasquill whether he considers his observations can be used in this way, if he has determined the amount of dew and, if so, what was the result.

Mr. F. Pasquill (*in reply — partly communicated*) : In reply to Dr. Penman's queries on the aspiration of psychrometers it has always been my impression that,

in general, controlled aspiration is a desirable feature. In particular cases I agree that the advantages in instrumental performance so gained should be weighed against the resulting disturbance of the natural structure of the air, and I may say that, in the present application, subsequent laboratory tests of a simple nature suggested that the latter effect was of negligible importance. In more difficult situations, such as that presented by measurements in the relatively stagnant air within crops, it is perhaps worth bearing in mind that a reduction in the required aspiration may be permitted merely by reducing the size of the wet bulb.

Dr. Penman's doubts concerning the basis of the equation for rate of evaporation should have been dispelled by Professor Sheppard's contribution. As Professor Sheppard has rightly pointed out, the equation need not be regarded as dependent on possibly controversial theoretical aspects and indeed may be looked upon as a deduction from observations including *measurements* of natural evaporation. These observations are only referred to briefly in the paper but have been discussed in full elsewhere (see references).

I am not clear on what measurements Dr. Penman bases his statement of a wind velocity effect different from that implied in Eq. (5) but, as I have emphasised in the paper, this equation refers strictly to evaporation and profile characteristics in *adiabatic flow over an aerodynamically rough surface*. It certainly cannot be expected generally to express the results provided by the evaporation tanks which have figured largely in the discussion. Furthermore I suggest that in the analysis of such results the influence of wind velocity alone may be masked by the tendency for the most marked departures from adiabatic conditions to be associated with the lightest winds.

Regarding the points raised by Mr. Lapworth, there is nothing in the basis of Eq. (5) which prohibits its application to rough surfaces other than grassland, though in considering tall crops, for example, extra care would need to be taken in the interpretation of the profile measurements and this is in fact a point requiring further observational study. In evaluating the equation satisfactorily the observational requirements are admittedly exacting, but at the present stage I cannot suggest any way of avoiding this without loss of accuracy in the final results. Dr. Sutcliffe enquired about the evaluation of dew deposition by the present method ; this could of course be carried out merely by integrating the appropriate sections of curves such as those at the bottom of Fig. 1. I should point out, however, that the observational justification of Eq. (5) has so far been confined to evaporation as distinct from condensation, though it is difficult to see any good reason why the equation should not apply equally to the latter, at any rate within the limits of approximation imposed by the neglect of departures from adiabatic conditions.

# REVIEWS

*Atmospheric Turbulence.* By O. G. Sutton. London (Methuen & Co. Ltd.) 1949. F'cap 8 vo. Pp. 107 ; 4 figures. 6/-.

Professor Sutton's contributions to the theory of turbulent diffusion of smoke and water-vapour from sources on the ground are well-known and have improved our understanding of these complex problems. His work has been made the basis of practical methods of computing the data required in chemical warfare, and has also been applied to questions of pollution by industrial smoke. More recently he has written on the mechanism of transfer of heat from the ground under conditions of vertical instability. Always stimulating and provocative, he can claim much of the credit for the healthy state in this country of research into the very important problem of turbulent diffusion in the atmosphere.

This little book by Professor Sutton can therefore be assured of reaching a wide and appreciative audience among meteorologists. Written in the author's usual lucid style, it binds together into a coherent account all the contributions which he has made to the subject. So far as the size of the book permits, it also provides a general account of the meteorology of the lower atmosphere (the book is concerned only with conditions in the lowest 100 m or so of the atmosphere — should not the title be more precise ?) and of turbulent motion, including a chapter on the effect of a variable density gradient on turbulence. A chapter is devoted to the usefulness and the limitations of the empirical " K-theory." This theory is based on the assumption that the turbulent transfer can be described by a transfer coefficient (which may vary with position) equivalent to the molecular coefficients of viscosity, heat conduction, etc., the value of the eddy transfer coefficient being obtained from a comparison with observations. The usefulness of the theory arises from Ertel's demonstration that the shearing stress is approximately constant in the lower layers, which permits the eddy transfer coefficient to be deduced from observations of the wind profile, in turn permitting the profile of any other transferable quantity such as matter to be deduced. The analysis which Professor Sutton devised to replace the K-theory is based essentially on the ideas of the mixing-length theory used by Prandtl and Kármán and others, and so a valuable chapter is given to a description of these ideas and the relevant results of wind-tunnel research. There is a brief mention of the statistical treatment of turbulence initiated by G. I. Taylor but Professor Sutton is on less certain ground here and the section is not well integrated with the remainder of the book.

One might cavil a little at Professor Sutton's choice of the Methuen Monograph series for the publication of this work. Admittedly the series is intended to supply readers with a compact statement of the modern position in each subject, rather than to be used as a series of text-books, but it surely must be assumed that the compact statement is about uncontentious, firmly-established results of previous research. I doubt whether even the author would claim that the theoretical work described in the book under review is (scientifically) satisfactory. There is, of course, a real need for vigorous and stimulating discussion — of the kind we have so often had from Professor Sutton — of ideas not yet in their final logical form, but such discussion should be within a framework which does not lend authority to those ideas. The space available in this book is so limited that the author has not included one quantitative diagram of observations — and yet his principal justification for introducing hypotheses and plausible functional relations is that they lead to results in agreement with the facts. Until these hypotheses are on a sound analytical basis, it should be the responsibility of the author to let readers see how good the reported agreement is.

Nor is it being too pedantic to insist that such a book should emphasize the basic principles of the subject. It is a *sine qua non* of a book about the motion of real fluids that the significance of the relevant dimensionless physical parameters such as the Reynolds number and the Richardson number should be explained clearly. In the reviewer's opinion Professor Sutton unfortunately does little to dispel the common misconception that the Richardson number has significance as a local quantity. In fact it is a reference parameter for a whole flow field, just as is the Reynolds number. The diversity of estimates of the critical value of the Richardson number arises partly from the fact that it has been defined in different ways and partly from the confusion between two different problems, *viz.* the growth of turbulence in a laminar flow and the suppression of turbulent motion. The reader who looks for precision of ideas and a clear understanding of the underlying physical principles will encounter several other sections which leave him unsatisfied. The fault is of course not wholly Professor Sutton's, since the subject of turbulence is quite literally in a mess. There is a danger that the persuasive style of the author may lull the uncritical reader into a state of happy ignorance of that fact.

G.K.B.

*Climatology—treated mainly in relation to distribution in time and place.* By W. G. Kendrew. Oxford (Clarendon Press) 1949. Pp. 383 ; illus. 30/- net.

The third edition of Kendrew's well known " Climate " has been so extensively revised as to be almost a new book, and the title has been made more appropriate. It forms an excellent introduction to modern climatology, mainly descriptive but with some account of the simpler physical principles involved. For the most part the treatment follows the time-honoured lines — Insolation and Temperature, Atmospheric Pressure and Winds, Water Vapour, Rain, Cloud, Sunshine and Visibility, Mountain and Plateau Climates, each divided into a number of short chapters. A chapter on Air Masses shows how the temperature at any one place is made up of the local balance of radiation and the effect of temperatures imported from afar. The discussions are lucid and readable and are illustrated by a large number of clear diagrams. Two minor criticisms are that the part played in the zonal distribution of radiation by the angle at which the sun's rays strike the ground might have been brought out more — it is barely mentioned — and that the " diagrammatic arrangement of pressure and winds on a rotating globe " (Fig. 38) is now quite obsolete and probably wrong anyway. The simple but artistic diagrammatic picture (Fig. 37) of winds and clouds by day and night over Lake Victoria, on the other hand, brings out the essential features very graphically and gave the reviewer much pleasure.

The two final parts deal with the " Weather of the Westerlies " and selected climatic types — The Sudan, Mediterranean and again the Westerlies as shown in British Columbia (maritime) and Manitoba (continental). The chapters on the Westerlies are more meteorological than the rest of the book, dealing with pressure systems, fronts, depressions and spells (types) of weather ; they serve a useful purpose in showing how the " climate " in temperate regions is made up of a succession of synoptic types. The few paragraphs on Periodicity are so superficial that they would have been better omitted. The reality of the 35-year Brückner cycle can hardly be said to be "well established" and it certainly cannot now be used as an argument for a 35-year period for normal values.

A feature of the book is the very fine series of sixteen photographic plates, mostly clouds but also flood, rime, snow and atmospheric pollution (which one is glad to see recognised as an important climatic — or should we say, " anti-climatic " — element). The photograph reproduced from *The Times* as Plate 4, showing cumulus clouds over the land on both sides of the Channel, is not only a pleasing picture, it is almost a lesson in meteorology by itself.

C.E.P.B.

*Hamburg, Meteorologisches Amt für Nordwestdeutschland.* Klimatologie des östlichen Teils des Mittelatlantischen Ozeans nach Schiffsbeobachtung, Hamburg, 1947. 39 × 39 cm. Pp. iv + 75 ; 129 charts. (Price not quoted).

This valuable atlas covers the area between 40° N. and the equator, from the coast of Spain and Africa to the western limit of the main sea route (15° W. in the north to 30° W. in the south). It is the first fruit of the analysis of some 60,000 Hollerith cards mainly for the years 1922-39 with some for 1912-14 and 1918-21, collected by the Deutsche Seewarte. Statistics of wind are analysed exhaustively — Baillie wind roses for each 5° square, direction, strength and steadiness of the main air currents, frequencies of calms and strong winds — gales. Other monthly charts include mean pressure, mean cloudiness, air temperature and that important element, air minus water temperature. Seasonal tables include visibility and state of sea, frequency tables of pressure, cloud amount, air and sea temperature, and extremes of air temperature. It is interesting to note that the latter has reached 30° C (86° F) off the Guinea Coast in November. The atlas ends with quarterly charts of percentage frequency of rain and haze. All the charts are very clear and illustrate the climatic features neatly — though occasionally a " wiggle " may not represent a real phenomenon. Dr. Kuhlbrodt, who edited the atlas, is to be congratulated on a most welcome piece of work.

C.E.P.B.

*Das Klima Griechenlands.* Alfred Philippson. Bonn (F. Dümmlers) 1948. Pp. 238 ; maps. 14.80 marks.

Professor Philippson, born in 1864, is now emeritus of the University of Bonn. As one of the most senior German geographers he is chiefly known for his authoritative text on the Mediterranean

lands of which the first edition appeared in 1904. His work on Greece began in 1889 with a series of papers resulting from his travels in that country in 1887-8 and in this latest work he returns to the studies of his youth. He is indeed to be congratulated in producing a systematic survey of the climate of Greece, incorporating the work of such well-known authorities as Mariolópoulos and Livathinós, which can be recommended. It is a good straightforward climatological text in the old-fashioned German style and such an assemblage of facts cannot but be useful. Students of climate will find plenty of food for thought ; for example some will note that the extreme minimum recorded at Athens from 1894-1929 was 22° F ; in scattered earlier records since 1840 the absloute minimum was 18.6° ; but in February, 1934 a minimum of 12.6° befel, far outside any apparent statistical expectation. One cannot but appreciate, too, the painstaking summary of the observations on mountain snow-cover at exceptional seasons gathered by the author during his travels between 1888 and 1893, which he partly repeated in 1934. The final chapter on ' Climate and Man ' is an interesting if somewhat determinist statement of many of the environmental factors in Greek life so well known to the author, who reveals some nostalgia for his early days " when the influence of America had not begun to appear." There is a good summary of views on climatic change ; Philippson joins others in the opinion that as regards the actual climate no significant change has occurred since classical times ; the effects of deforestation and the like are another matter. Although there is little evidence of observational material since 1935 the book can be welcomed.

G.M.

*Magnetism.* By David Shoenberg, Ph.D. London (Sigma Books Ltd.) 1949. 8vo., Pp. 216 ; 8 plates ; 56 figs. 10*s.* 6*d.*

The author of this book has attempted a very difficult task, that of explaining a great range of complex and closely inter-related facts and theories of magnetism (and electricity) to the lay reader. He has done so with considerable success though the intrinsic difficulties of the subject have compelled him to state the results of agreement and calculation here and there, without pretending to make simple what is inevitably complicated. Besides giving a general account of the fundamentals of magnetostatics and the magnetic fields of electric currents, he gives a good survey of the modern theories of the magnetic properties of matter, that is, of the various particular kinds of magnetic matter. He also indicates how these properties are made use of in practical applications of many kinds, and moreover includes a chapter on the magnetism of the earth, which should be of particular interest to many meteorologists.

The book is well illustrated with plates and numerous figures and contains a useful Bibliography and Glossary-Index.

S.C.

*Allgemeine Meteorologie.* By Dr. Hellmut Berg. Bonn (Ferd. Dümmlers Verlag) 1948. 20·5 × 14·5 cm. Pp. 337 ; 86 figs. (Price *not* stated on the book but quoted from a list received in the Meteorological Office Library from a Hamburg bookseller — 14·60 DM.)

Professor Berg states in the preface that his new book is an introductory treatise for students of meteorology and workers in allied sciences.

The book is divided into 19 sections, of which the first 13 are concerned with meteorological elements and phenomena as observed at one place and time, and the remainder with movement and the relations between phenomena at different places and times. All aspects of meteorology including optical and acoustical phenomena are dealt with to a greater or lesser extent and a sketch of terrestrial magnetism is included as well.

The physics and mathematics used are scarcely of intermediate science standard. Professor Berg includes much elementary physics which is rather surprising in a book not intended for the general reader but for serious students. The readers to whom it is professedly directed should know their basic physics before commencing meteorology.

The author has written very good sections on clouds, atmospheric optics, atmospheric electricity and magnetism and " statical " meteorology (stability, tephigram, etc.). On the whole, however, the book does not give a well balanced view. Such important subjects as fog and turbulence receive very brief treatment. Synoptic and dynamical meteorology are, by comparison with optics, very inadequately described. Though the term " synoptic chart " is defined there is no example of

one. Surely a book on general meteorology should provide some actual synoptic charts of typical weather situations. Thermal winds and the uses of contours of pressure surfaces are not mentioned. No doubt the difficulties of obtaining foreign literature in Germany since 1939 account for such omissions as the absence of reference to the Dobson-Brewer frost-point hygrometer and the important low temperature humidity measurements made with it and to the important practical matter of the refraction and reflection of high-frequency radio waves in the lower troposphere.

The price is high for a book of this size, poorly bound in manilla covers and with only line-drawing illustrations.

G.A.B.

*Otkrytiye Kamchatki i ekspeditsii Beringa (The Discovery of Kamchatka and Bering's expeditions).* By L. S. Berg. (4th edition). Moscow, Leningrad (Izdatel'stvo Akademii Nauk SSSR [Publishing House of the Academy of Sciences of the USSR]) 1946. 8vo. Pp. 379; illustrations and maps. 30 roubles.

*Ocherki po istorii russkikh geograficheskikh otkrytiy (Studies in the history of Russian geographical discoveries).* By L. S. Berg. Moscow, Leningrad (Izdatel'stvo Akademii Nauk SSSR [Publishing House of the Academy of Sciences of the USSR]) 1946. 8vo. Pp. 358; illustrations and maps. 23 roubles.

The President of the Soviet All-Union Geographical Society has given us two scholarly contributions to the history of Russian exploration.

*The Discovery of Kamchatka and Bering's expeditions* was first published in 1924 and is now regarded as a standard work. It covers a fascinating and difficult period. Bering's second expedition (1733-43), to which two-thirds of the book are devoted, was an extensive undertaking. It included not only Bering's own voyage to the American coast, but also a voyage to Japan by one of his lieutenants, and explorations — collectively known as the Great Northern Expedition — of much of the northern coastline of Siberia by four separate teams. No contemporary account of these remarkable happenings was available for many years because the whole expedition was conducted under conditions of secrecy. Berg has therefore had to compile his account largely from archive sources. As it happens, English readers have already been provided with an account, no less detailed than Berg's, of Bering's own voyages. This is *Bering's voyages* by F. A. Golder (two vols., New York, 1922-25), a book to which Berg pays a well-deserved tribute. While Golder provides a fuller picture of the voyages in the north Pacific by allowing Bering's log books and the journal of the naturalist Steller to speak for themselves, Berg provides a great deal of additional information on the historical background, particularly to the exploration of Kamchatka, and gives accounts of all the subsidiary voyages under Bering's nominal control. To anyone interested in the history of exploration this is an important book.

*Studies in the history of Russian geographical discoveries* is a heterogeneous collection of essays designed principally to promote the interest of the Soviet citizen in the achievements of Russian explorers and geographers. There are essays on the polar expedition planned by Lomonosov, the remarkable Russian scientist, grammarian and author of the eighteenth century; on early maps of the Caspian Sea as sources of information on fluctuations in its level; on the journeys of the traveller N. N. Miklukho-Maklay to New Guinea in 1871 and 1883, and of A. I. Voyeykov, the climatologist and geographer, to Japan in 1876. There are biographical sketches, or aspects of the biography, of Ye. F. Zyablovskiy (1765-1846), the first professor of geography at St. Petersburg university; P. P. Semenov-Tyan-Shanskiy and N. M. Przheval'skiy, nineteenth century explorers of Tyan-Shan and central Asia; and D. N. Anuchin, a leading personality in geographical circles in Moscow at the turn of the last century. These are the great figures in Russian geography, and little has been written about most of them even in Russian. The book also contains other essays on Russian exploration. Each chapter is the result of careful research among sources which are largely unobtainable outside Russia. It is this aspect of the work which will make most appeal to English readers.

Both books are quite well produced. A really quite successful attempt has been made to make the illustrations comprehensible and the maps legible.

T.E.A.

*Some Recent Researches in Solar Physics* (*Cambridge Monographs on Physical Subjects*). By F. Hoyle. Cambridge (University Press) 1949. Pp. 134 + xi. 12*s.* 6*d.*

The aim of the new series of *Cambridge Monographs* is to present the results of recent research in a specified branch of physics, and the authors of the volumes already announced are all active workers in the field about which they write. Meteorologists will be gratified that two of these titles refer to atmospheric phenomena, whilst solar-terrestrial relationships are the subjects of several sections of the present monograph.

The author's preface states that the first draft was written in 1947, but brief notes in an appendix refer to developments since that date. The monograph claims to " provide a concise account of the present state of solar physics " ; in fact it is almost exclusively concerned with the chromosphere and corona, with only brief references to electromagnetic effects below the photosphere. Two chapters are devoted to an admirably concise outline of the main observed facts, though some may think that the doubts concerning the existence of the general solar magnetic field are perhaps too lightly dismissed. The structure of corona and chromosphere are then discussed on lines developed by the author and his collaborators in which the accretion of interstellar matter, hydrogen with a small content of heavy elements, plays the major part. There follows a chapter on electromagnetism in the highly conducting fluid which forms the solar material in the layers considered, with applications to the mechanism of prominences and flares. References to solar and terrestrial relationships are confined to discussion of corpuscular emission from the sun in the light of the accretion theory (with mention of a highly speculative theory of the origin of the earth's magnetic field), and of certain ionospheric problems. The final chapter is concerned with the emission of radio waves in normal conditions and from solar flares.

With so much material crowded into one small book one cannot expect easy reading, and much study would be needed to derive full benefit. The speculative nature of some of the arguments, and the wide range of the subjects involved may encourage some to make such a study, and perhaps to engage in controversy with the author. If that shoud be so both author and editors will consider their work well done for this is surely the main function of such monographs.

The production of the book in no way falls below the very high standards of its publishers.

G.D.R.

*Oscillations of the Earth's Atmosphere.* By M. V. Wilkes. Cambridge (University Press) 1949. 8vo. Pp. 74 ; 23 figures, 9 tables. 12*s.* 6*d.*

The forced oscillations of the atmosphere caused by gravitation (atmospheric tides) and varying insolation are the subject of Dr. Wilkes's monograph *Oscillations of the earth's atmosphere.* The account is fairly well balanced, roughly equal amounts of space being devoted to empirical analysis and mathematical theory. The empirical analysis includes some of the recent work on surface data (especially the statistical work of Chapman) and also some very recent work on behaviour at ionospheric levels. On the theoretical side Dr. Wilkes first develops the relevant differential equations and then proceeds to discuss certain " sample " solutions obtained by numerical analysis. Needless to say, in such a short work it is impossible to be comprehensive (the problems involved are at least as complicated as the closely related, but distinct, problems of the oceanic tides) and Dr. Wilkes pays special attention to the semi-diurnal component oscillations. In the case of the moon this is the largest component and the only one as yet observed. In the case of the sun the semi-diurnal component is very much larger than that of the moon (in spite of smaller gravitational excitation) and is even larger than the solar diurnal component (in spite of smaller thermal excitation). The most obvious explanation, that the atmosphere " resonates " (*i.e.*, has a free period) very close to half a solar day, was at one time suspect but the recent work of Pekeris and Weekes and Wilkes, described in some detail, shows the resonance theory to be at least not inconsistent with available observational material. At the same time it is clear that the last word has not yet been said. Dr. Wilkes indicates some questions (*e.g.*, the relative importance of thermal and gravitational excitation of the solar semi-diurnal component) which have not yet been answered and the critical reader will note many more (*e.g.*, the question of phase-lag and seasonal or annual variation). In future editions it might be possible to devote more space to such problems. A simple physical account of the mechanism of excitation (especially thermal excitation) would be an advantage, as would a few diagrams indicating wave structure.

E.T.E.

*Ultra-violet and daylight rays in the solar cycle.* By J. R. Ashworth, D.Sc. Liverpool (University Press) 1949. 8vo. Pp. 73 ; 11 figures, 11 tables. 6*s*.

It is much easier to measure either the duration of sunshine, or the instantaneous brightness of a white surface, than the total energy, in one wavelength or a group of wavelengths, received each day from the sun and sky. To Dr. Ashworth must go full credit for having produced an instrument for doing this which, for 17 years at any rate, has stood the test of time. His ingenious photographic method probably has most of the accuracy attainable without elaborate photometry, and the instrument is simple and robust.

One proviso must, however, be made. In its present form the instrument " does not register light over a range of more than 30° or 40° from the zenith when it is supported in a horizontal position." Near midsummer the sun passes within the cone of acceptance each day, but for the rest of the year it does not. Probably it would be better to reduce the half-angle of acceptance artificially to 20° or less, or to tilt the instrument towards the north, instead of exposing it horizontally.

When he discusses the results of systematic observations with his instrument, Dr. Ashworth invites his readers to swallow a very tall story. During the 11-year solar cycle ultra-violet radiation increases and visible daylight diminishes with increasing sunspot activity, and the ratio of ultra-violet to visible is twice as great at sunspot maximum as at the minimum ! There may be room for argument about the variation in ultra-violet in the small part of the band considered by the author but the variation in visible daylight could hardly have gone unnoticed by the astronomers who have been observing the sun for very many years.

Having produced a deal of evidence in favour of this assertion, Dr. Ashworth explains it by assuming that the ionized regions of the upper air absorb both visible and ultra-violet radiation. This, too, is a phenomenon which would have been observed long ago by physicists and meteorologists as well as astronomers, if it occurred on one tenth the scale Dr. Ashworth assumes of it.

A.R.M.

*Climatic and Weather Exercises.* By W. G. V. Balchin and A. W. Richards. London (Harrap and Co., Ltd.) 1949. Pp. 79 ; 46 maps and diagrams. 6*s*.

At present elementary text-books of meteorology are fairly numerous, but as few of them are suitable for adoption as a school text-book, the appearance of this volume is particularly welcome. It embraces the meteorology normally included in school geography courses, and it is indeed pleasant to see a whole volume devoted to it. Both the climatological and the synoptic aspects are treated with great clarity in an elementary manner, but the outstanding feature of the book is the excellence of the practical work outlined, both observational and cartographical. Suggested observational work includes temperature, humidity, rainfall, and so on, but it does not stop there : it enters the realm of microclimatological investigation in which the authors are already noted ; it includes the use of such things as petri dishes, and we are even given a glimpse of a squad of pupils mounted on bicycles and armed with sling thermometers, to be slung at some pre-arranged place and time. The classroom work is presented in a way likely to be very interesting to the pupils, and it is good to see that when statistical data are given, figures from the Meteorological Office have been employed. The processes underlying climatic differences are given prominence, and are skilfully introduced, usually via practical work.

Although the work is not quite free from blemishes, such things are not numerous. One feels that the observational work on pressure has not received enough notice ; moreover sunshine has been omitted altogether. A number of reasonable Jordan type recorders can easily be made from large cocoa-tins, lenses, wood and wax, together with a supply of daylight papers. One would also have liked to see clouds dealt with more fully, with some photographs. The reviewer was sorry to see (p. 73) a reference to raingauges made from a bottle and funnel ; such things, even if identical in form, react differently in different situations, and are useless. Again (p. 30) the rim being a foot above the ground does not prevent the ingress of splashed drops. The use (p. 26) of steam and water-vapour as synonymous terms is not correct ; haze (p. 28) is not a less dense form of mist ; and the symbol for snow (p. 52) is given as an 8-pointed star instead of one with six points. On the station model on p. 50 the visibility should be in furlongs. On p. 71 there is a picture of a Bilham Screen with the caption " Stevenson Screen " underneath. On p. 15 the authors have made a dubious statement in trying to simplify the explanation of why there is a lapse-rate of temperature. Only one misprint was detected, a mis-spelling of Rhayader on p. 20 and on the map opposite (similar occurrences with Welsh names are lamentably frequent in many books). It is good to see that the

authors appreciate the gentle climate of the North Wales coast, which is often overlooked ; but it is surprising to see North Wales placed in the "South-West" on this account (p. 68). The book is nevertheless excellent, and it is to be hoped it will receive the welcome it deserves. Perhaps when school meteorology starts as a subject on its own the authors will consider writing a text-book of the same standard, with the inclusion of some mathematics and physics.

S.E.A.

*Ultraviolette Strahlen.* By A. E. Herbert Meyer and E. O. Seitz. Berlin (Walter de Gruyter) 2nd Edition, 1949. Pp. 390 and XVI ; 20 plates. 34 DM.

This book is of a type more common in the German than the English language, being concerned with the technology of ultra-violet rays and their applications in a number of sciences and industries, and making every effort to be exhaustive in treatment. Some idea of the immense industry of the authors may be gained from the fact that the bibliography, a most valuable feature, contains just short of 1,500 entries, a good proportion being to post-war work, with a reasonable proportion of American and British authors.

The first chapter is devoted to sources of U.V. and includes the subject of most direct meteorological interest, a discussion of the U.V. content of Sun- and Sky-light. This is followed by a section on measurement, including spectrophotometric methods and a valuable discussion of filters. The final chapter is devoted to applications of U.V. in physics, chemistry, biology and medicine, and an appendix contains tables mainly concerned with the performance of various types of lamp, and the details of filters. The text contains 261 figures and 58 tables.

In a work which touches on such widely divergent subjects as the ultra-violet content of the light in large cities, the amplification of photocell currents, the detection of forgery by fluorescence microscopy, "sun-baths" in large factories, and the sterilization of milk, to name but a few, it is not to be expected that the specialist in one narrow field will always find the most up-to-date information. As a reference and source book for the non-specialist, however, the work is of considerable value, and does not appear to suffer from the inclusion of out-of-date work merely for the sake of historical completeness (or indeed merely for the sake of length, as is the case in some volumes of its type). It might also serve to give the specialist a more balanced outlook on his subject, and is particularly recommended to the non-physicist interested in applications of U.V. light. It cannot claim much professional interest for the meteorologist, but he would be well advised not to answer questions concerning ultra-violet in the atmosphere without consulting it, and he will no doubt derive a melancholy satisfaction from contemplating the lengths to which some of the Ruhr industrialists were prepared to go to replace on the skins of their workmen the dose of U.V. which the smoke of their factories removed from the sunlight.

It remains to be said that the printing, binding and design of the book are good by any recent standards, and far in advance of anything the reviewer has seen in post-war German books.

G.D.R.

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When Inwards died in 1937, at the age of 97, he left the copyright in *Weather Lore* to the Royal Meteorological Society. By then the three 19th-century editions had all been long out of print and second-hand copies were scarce and costly. The Council therefore decided, soon after the war, to re-issue the book at about the time of the Society's centenary celebrations during the spring of 1950. This, it was felt, would be as the author would have wished. The task of revising and editing the material was entrusted to Mr. E. L. Hawke, M.A., who had already formed a collection of ancient and more modern weather adages lacking from Inwards's compilation.

This fourth edition, now in course of printing by Messrs. Rider & Co. for publication a few weeks hence, is thus even more comprehensive than its forerunners. In addition to providing origins or explanations for many of the sayings and rules cited, Mr. Hawke has contributed a prefatory chapter on the nature and significance of weather lore and a biographical note on Richard Inwards, an outstanding figure of the Victorian era and a one-time pupil of John Ruskin. Reproduced in the volume are a number of the finest cloud photographs from the famed collection of the late Mr. G. A. Clarke, F.R.P.S. An up-to-date version of Inwards's well-known schematic diagram of cloud forms and heights is also included.

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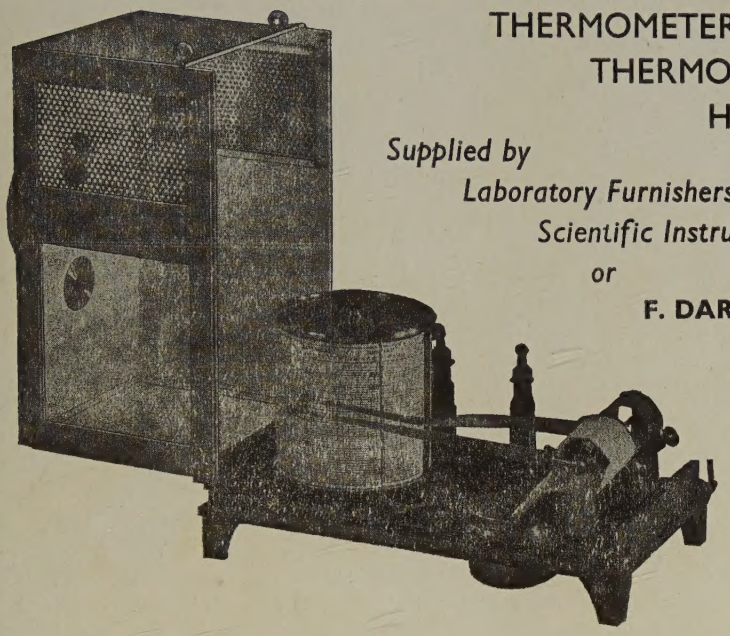

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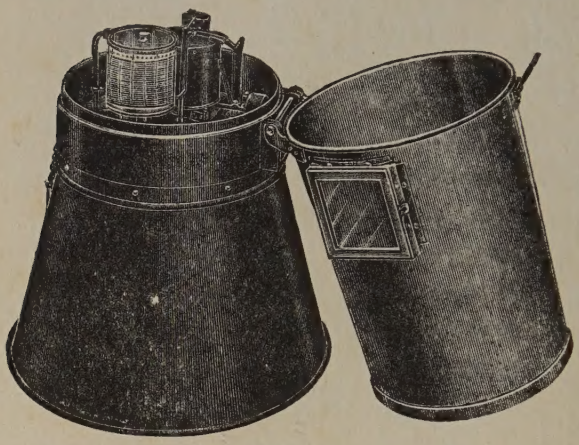

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